Lagrangian Chaotic Mixing in Resistive Drift-Wave Turbulence in Fusion Plasmas

A. L. Piragibe[1,2] , R. A. Miranda[1,2] , A. B. Schelin[2] and J. L. Ferreira[2]

[1] *UnB-Gama Campus, University of Brasília (UnB), Brasília DF, Brazil*
[2] *Plasma Physics Laboratory, Institute of Physics, University of Brasília (UnB), Brasília DF, Brazil*

Abstract: The Lagrangian chaotic mixing properties of electrostatic resistive drift-wave turbulence is reported in numerical simulations of a tokamak plasma modelled by the modified Hasegawa-Wakatani (MHW) equations. We focus on two different regimes, namely, a regime dominated by turbulent patterns, and a regime dominated by zonal flows. A transition between these two regimes occurs by changing the value of a control parameter related to adiabaticity, and serves as a simplified model of the low-to-high confinement (L-H) in tokamaks. Lagrangian coherent structures (LCS) are detected by computing the finite-time Lyapunov exponent (FTLE) of the computed velocity field, and the statistics of the chaotic mixing of the two regimes are compared. These results can contribute to the understanding of turbulent transport processes in fusion plasmas.

Keywords: fusion plasmas, L-H, MHW, FTLE, LCS, PDF, PDFs

1. Introduction

In fusion plasmas, one dominating problem is the turbulent processes that presents themselves in the radial transport at the edge of a tokamak [1]. Understanding the dynamics of the turbulent radial flux of particles and heat, in magnetized plasmas, are vital to fusion research, and can lead to improvements in the confinement properties of fusion devices, such as the tokamak [2]. To determine the confinement properties of the overall plasma in the bulk region, numerical simulations can be used to model this behavior. The Hasegawa-Wakatani equations allow for an understanding of the radial transport in two-dimensional numerical simulation of electrostatic resistive drift-wave turbulence.

We perform numerical simulations to better understand a phenomenon, in fusion plasmas, called low-to-high confinement (L-H) transitions. This happens spontaneously when plasma is in a low confinement stage, known as turbulent flow, and transitions into a turbulence-suppressed regime, called zonal flow. The zonal flow is related to the high confinement stage. This transition is of outmost importance, because it enhances the confinement and studying L-H transition allows for confinement improvement [3]. In addition, we compute the finite-time Lyapunov exponent (FTLE) used to detect the Lagrangian coherent structures (LCS), and better characterize the chaotic mixing during the L-H transition.

2. Computational Methods

Numerical simulations are performed, using a simplified model of a tokamak plasma, from the modified Hasegawa-Wakatani (MHW) equations, which take into consideration the effect of the zonal component [3]:

$$\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} = \alpha(\tilde{\varphi} - \tilde{n}) - D\nabla^4\zeta \tag{1}$$

$$\frac{\partial}{\partial t}n + \{\varphi, \zeta\} = \alpha(\tilde{\varphi} - \tilde{n}) - \kappa\frac{\partial \varphi}{\partial y} - \nabla^4 n \tag{2}$$

where the physical setting of the model is set to be in a constant magnetic field equilibrium $B = B_0\nabla z$, and $n_0 = n_0(x)$ which characterizes the nonuniform density of the edge region of a tokamak plasma. The parameters in equations (1) and (2), $\{a, b\}$ is the Poisson bracket, n signifies the density fluctuations, while the ion vorticity $\zeta \equiv \nabla^2\varphi$ is a 2D Laplacian dependent on the electrostatic potential (φ). The background density $\kappa \equiv (\partial/\partial x)\ln n_0$ is a constant with an unchanging exponential profile, whereas D is the dissipation coefficient. Lastly, α is an adiabaticity operator that is turned into a constant coefficient when set in this physical setting [3].

The MHW equations are yielded when subtracting the zonal components from the resistive coupling term, turning $\alpha(\varphi - n)$ into $\alpha(\tilde{\varphi} - \tilde{n})$. The velocity field equations are obtained from the electrostatic potential (φ) [3]:

$$v_x \equiv -\frac{\partial\tilde{\varphi}}{\partial y} \tag{3}$$

$$v_y \equiv -\frac{\partial\tilde{\varphi}}{\partial x} \tag{4}$$

The particle density flux (Γ_r), PDF, is a correlation between the particle density (n) and radial velocity (v_r = -$\partial\tilde{\varphi}/\partial y$) [2]:

$$\Gamma_r = \ < n v_r > \tag{5}$$

The chaotic mixing properties are characterized by computing the finite-time Lyapunov Exponent (FTLE) and observing the Lagrangian coherent structures (LCS) that are formed from the velocity fields. The LCS are defined as ridges in the FTLE fields. These ridges are special gradient lines of the FTLE field that are transverse to the direction of minimum curvature [4].

$$\sigma_{t_0}^{t_0+\tau}(\vec{x}) = \frac{1}{|\tau|} ln\sqrt{\lambda_1} \tag{6}$$

where Eq.6 represents the largest FTLE with a finite integration time τ and its absolute value permits the use of both negative and positive integration times τ. The positive integration time is a forward-time integration that reveals a repelling LCS [4].

3. Simulation Results

The MHW equations are solved using a finite differences method with a grid of 256x256, using a Fortran code. The FTLE is computed using a C code with a grid of 1024x1024. Afterwards, MATLAB is used to visualize the results from both MHW and FTLE, and to also calculate the values of the PDF.

Figure 1 shows the electrostatic potential φ using the solutions from the Fortran code simulation, which solved the MHW equations applying finite differences method. These solutions were imported into MATLAB using a 256x256 grid resolution.

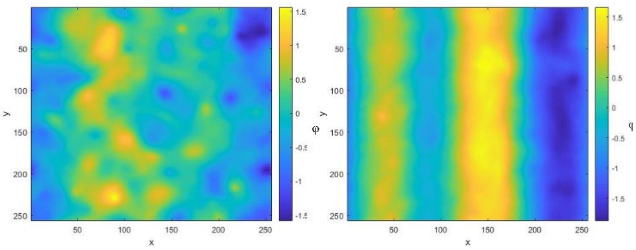

Fig. 1. The electrostatic potential in the (Left) Turbulent regime and (Right) Zonal flow.

A transition between these two regimes occurs by changing the value of a control parameter related to adiabaticity (α) [3] and serves as a simplified model of the L-H in tokamaks, the left panel in Fig. 1 represents low confinemen and the right being high confinement.

Figure 1 shows that φ becomes a zonally elongated structure in the zonal flow. This is due to the Kelvin-Helmholtz instability of the drift waves, which effectively suppresses the drift wave activity [3], causing it to become a high confinement zone known as the zonal flow.

Fig.2 shows detected LCS using the FTLE ($\sigma_{t_0}^{t_0+\tau}$) from the velocity fields. These solutions are then imported into MATLAB to create the images shown in Fig. 2 with a 1024x1024 grid resolution.

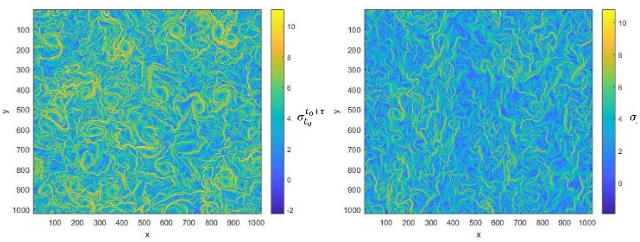

Fig. 2. The FTLE in the (Left) Turbulent regime and (Right) Zonal flow.

When analyzing the images, Fig. 2 exhibits a much more detailed understanding of where the transport barriers (LCS) are. These barriers are displayed using a color gradient, where the stronger barriers are in yellow, while the weaker are in blue. When comparing both images (in Fig. 2), it reveals that the higher number of strong barriers are, in fact, in the turbulent regime (left), as to be expected.

While contrasting Fig. 1 and Fig. 2, Fig. 2 has a clearer visualization of where the barriers are formed. As the images from Fig. 1 overlap into Fig. 2, it can be observed that not every contour plots from Fig. 1 is actually a vortex. This demonstrates that FTLE is a better at detecting vortices when compared to a visual inspection of the electrostatic potential.

Table 1. The results of the PDF (Γ_r) calculations

Turbulent Regime	1.4110
Zonal flow	-0.07577

Table 1 shows the computation of the radial flux (Eq. 5) for the turbulent and the zonal flow regimes, for the entire simulation domain. It is evident that the turbulent regime displays a higher value of radial flux than the zonal flow, which is expected since the elongated patterns of the zonal flow act as transport barriers of the flow.

Figure 3 shows the probability distribution functions (PDFs) of the FTLE shown in Fig. 2. Broad PDFs are related to heterogeneous mixing [5]. From Fig. 3, it can be determined that the turbulent regime has a more heterogeneous mixing than the zonal flow regime, because the turbulent PDFs is broader than the corresponding PDFs of the zonal flow. This is to be expected, since the zonal flow is a regime of high confinement, suppressing turbulent transport [3].

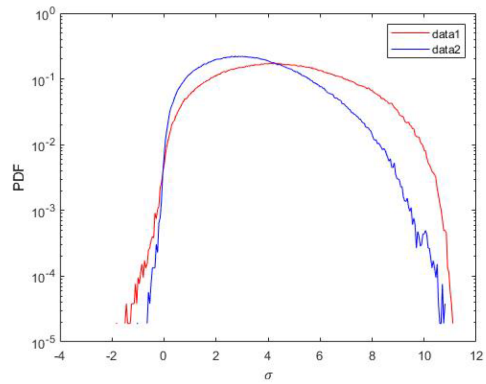

Fig. 3. PDFs of turbulent regime (data 1) versus zonal flow (data 2).

4. Conclusion

In conclusion, we performed numerical simulations of the modified Hasegawa-Wakatani equations in two regimes, namely, a turbulence-dominated regime and a zonal flow regime. The turbulent mixing properties of the flow was characterized by applying the finite-time Lyapunov exponents, which is a common tool for the understanding of the Lagrangian properties of turbulent fluids. By constructing probability distribution functions (PDFs) we conclude that the turbulent regime displays a more heterogeneous mixing than the zonal flow regime, which is in agreement to the fact that the zonal flow is related to a high-confinement regime. The techniques applied here can be useful for the understanding of drift-wave induced turbulence in tokamak plasmas.

5. References

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