A SIMPLIFIED UNIFIED THEORY OF ARCS AND THEIR ELECTRODES

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Abstract

A recently developed arc-electrode theory has been simplified so that electrode and arc regions are treated together. Points at the surface of the electrodes are treated in a special way to account for electrode effects. The theory is used to make predictions of arc and electrode temperatures, and arc voltages, for arcs in argon as a function of current from 50 to 400A. Maximum arc temperatures and volt-ampere characteristics are compared with experimental results.

Introduction

Our earlier theoretical treatment of electric arcs and their electrodes [1-3] has been simplified in a number of ways as follows: (1) In the cathode sheath we have omitted the electron ionization term due to the electric field and we do not solve the electron energy balance equation for the sheath. Only the electron continuity equation involving ambipolar diffusion, thermal ionisation and recombination is solved. (2) Previously solutions were needed for an arc region, two electrode regions and two sheath regions, and then further iterations were necessary to satisfy boundary conditions between all regions. Now we consider the arc-electrode system as a whole, but with special consideration for points on the electrode surface. (3) We use a simplified boundary condition at the cathode based on the theoretical thermionic current. (4) Radiation heating of the electrodes by the arc has been neglected, thus avoiding an integral to obtain radiation flux.

With these simplifications, computation times are reduced by about a factor of 100, compared with those of [1-3]. The new method makes possible the prediction of arc and electrode temperatures, axial and radial plasma velocities, and plasma pressure, all in two dimensions, as well as arc voltage, for any arc current and electrode geometry. It is assumed that the cathode is a thermionic emitter. The material properties of the gas and the two electrodes are required as a function of temperature.

Basic Equations

1. Arc Column. The plasma is assumed to be in local thermodynamic equilibrium and the flow is assumed to be laminar. The basic equations of conservation of mass, enthalpy, radial momentum and axial momentum, are respectively:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{\partial}{\partial z}(\rho v_z) = 0 \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r h) + \frac{\partial}{\partial z}(\rho v_z h) = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{rk}{c_p}\frac{\partial h}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{k}{c_p}\frac{\partial h}{\partial z}\right) + \frac{j_r^2+j_z^2}{\sigma} - U \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r^2) + \frac{\partial}{\partial z}(\rho v_r v_z) = -\frac{\partial P}{\partial r} - j_z B_\theta + \frac{1}{r}\frac{\partial}{\partial r}\left(2r\eta\frac{\partial v_r}{\partial r}\right) + \frac{\partial}{\partial z}\left(\frac{\eta\partial v_z}{\partial r} + \frac{\eta\partial v_r}{\partial z}\right) - 2\eta\frac{v_r}{r^2} \tag{3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_z v_r) + \frac{\partial}{\partial z}(\rho v_z^2) = -\frac{\partial P}{\partial z} + j_r B_\theta + \frac{\partial}{\partial z}\left(2\eta\frac{\partial v_z}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\eta\partial v_z}{\partial r} + \frac{r\eta\partial v_r}{\partial z}\right) + \rho g \tag{4}$$

Equations (1) to (4) define the enthalpy, h, the pressure, P, and the axial and radial velocities, v_z and v_r, of the arc column, as a function of the radial and axial coordinates r and z. The material functions are the thermal conductivity, κ, the electrical conductivity, σ, the specific heat, C_p, the density, ρ, the radiation emission coefficient, U and the viscosity, η, all of which are required input as a function of temperature for the ambient pressure of the arc; g is the acceleration due to gravity. The current densities j_z and j_r are evaluated from Ohm's Law, $j = \sigma E$, where the electric field, E, is obtained from $E = -\nabla\phi$ where the electric potential, ϕ, is obtained from a solution of the current continuity equation, i.e. $\nabla.(\sigma\nabla\phi) = 0$. The magnetic field, B_θ, is obtained from Maxwell's equation: $(1/r)\partial(rB_\theta)/\partial r = \mu_o j_z$, where μ_o is the permeability of space.

2. Electrodes. Equation 2 is also applied for the regions within the electrodes. The electrodes are assumed to remain solid; hence there is no convection within the electrode regions and equations 1, 3 and 4 are not needed. Within the electrode volumes, values of thermal and electrical conductivity appropriate to each electrode are used.

3. Electrode Surfaces. Calculations at points on the electrode surface need to include

special processes occurring at the surfaces.

(1) Energy Fluxes. We account for energy losses by thermal radiation from the hot electrode, cooling or heating by the electrons leaving or entering the solid and heating by ion bombardment. Heating of the electrodes by radiation from the arc is neglected. For the cathode, the additional energy flux, F, is

$$F = -\varepsilon a T^4 - j\psi + j_i V_i; \qquad (5)$$

ε is the emissivity of the surface, T is the surface temperature, ψ is the work function, j_i is the ion current density, a is the Stefan Boltzmann constant, and V_i is the ionization potential of the plasma. The term in ψ represents the loss in energy from electrons leaving the cathode and overcoming the work function potential. The ion current density, j_i, is assumed to be $j - j_R$ at the cathode surface, where j is the cathode surface current obtained from the current continuity equation and j_R is the theoretical electron current due to thermionic emission obtained from $j_R = AT^2\exp(-\psi e/kT)$; e is the electronic charge, k is Boltzmann's constant and A is the thermionic emission constant for the surface of the cathode. If j_R is greater than j we take j_i to be zero. At the anode surface we make the term in ψ of Eq. 5 positive as electrons heat the anode due to the work function potential. We assume that there is no ion current and hence no ion heating at the anode.

(2) Effective Electrical Conductivity. For the mesh interval adjacent to the electrodes, the temperatures are less than 4000K and the equilibrium electron density and electrical conductivity are near zero. However, in practice, non equilibrium effects such as thermionic emission and ambipolar diffusion can make these regions highly conducting. To account for these effects an effective electrical conductivity has been calculated for these mesh intervals.

First, the electron continuity equation is solved for the electrode sheath mesh interval ie

$$\nabla.(D_A \nabla n_e) + S_o - \gamma n_e^2 = 0; \qquad (6)$$

D_A is the ambipolar diffusion coefficient [2] for the local temperature given by $2kT\mu_i/e$, μ_i is the ion mobility, n_e is the electron number density and γ is the electron ion recombination coefficient. A source term, S_o, representing changes in electron and ion densities due to thermal ionization, is defined as $S_o = \gamma\, n_{eq}^2$, where n_{eq} is the equilibrium plasma value of electron density for the local plasma temperature. Then solutions for n_e in the sheath will be the equilibrium values if there is no diffusion. For the solution of equation 6 boundary conditions are needed for n_e. On the plasma side of the mesh interval the equilibrium plasma value is used. On the electrode side we take $n_e = j_R/ev_t$, where j_R

is the thermionic emission current and the thermal velocity v_t is obtained from $mv_t^2/2 = 2kT$; m is electron mass.

Second, using the values of electron density as a function of distance within the sheath, the effective electrical conductivity of the sheath, σ_{eff}, is derived. A generalised form of Ohm's Law [2] is used:

$$j = n_e e / \{ n_0 / n_T \mu_e E + 2e n_e n_{eq}(T) / n_T \sigma E \}; \qquad (7)$$

n_0 is the equilibrium neutral particle density, μ_e is the electron mobility and $n_T = n_0 + 2n_e$ is the total particle density. This expression reduces to $j = \sigma E$ in the plasma where $n_0 = 0$ and to $j = n_e e \mu_e E$ in a cold gas where $n_0 \sim n_T$ and $n_{eq} \sim 0$. Eq. 7 is used to derive E as a function of position within the sheath for a given j and thus by integration the potential drop, V_s, across the mesh interval is obtained. Then $\sigma_{eff} = js/V_s$ where s is the mesh thickness. For the plasma adjacent to the anode we have simply used the local equilibrium electrical conductivity [2].

(3) Effective thermal conductivity. We have used the function $S(T) = \int \kappa \, dT$ within the electrode sheaths to avoid large numerical errors resulting from the large temperature differences that can exist across the sheath. Use this integral S, accounts for maxima in κ that can occur within this temperature interval.

Results

Calculations are made for free burning arcs in argon having the electrode configuraton of Fig. 1, which is the same as in [1-3]. The cathode is of thoriated tungsten and the anode of water cooled copper. The material functions used for argon, thoriated tungsten and copper were the same as used in [1-3]. Fig. 1 shows the predicted temperatures of a complete arc system in argon at 1 bar with a current of 200 A. The experimental data of Haddad and Farmer [4] for an arc under the same conditions are also plotted in Fig. 1. A uniform mesh is used in the regions of high current density at both electrodes, this minimum mesh size being 0.05 mm in the z direction with 90 points. There are 60 points in the radial direction.

Theoretical predictions for 200A, compared with those of [1] given in brackets are: maximum arc temperature 24000K (23700K), maximum cathode temperature 3800K (4000K) and arc voltage 12.7V (12.9V). The differences between these two sets of results are not significant and are partly due to differences in grid size. The calculations indicate that Ohmic heating in the cathode sheath is small.

points, were taken with the experimental system described in [5]. The theoretical predictions are in fair agreement with the experimental results except at low currents, where predictions of maximum arc temperatures and arc voltages are slightly low.

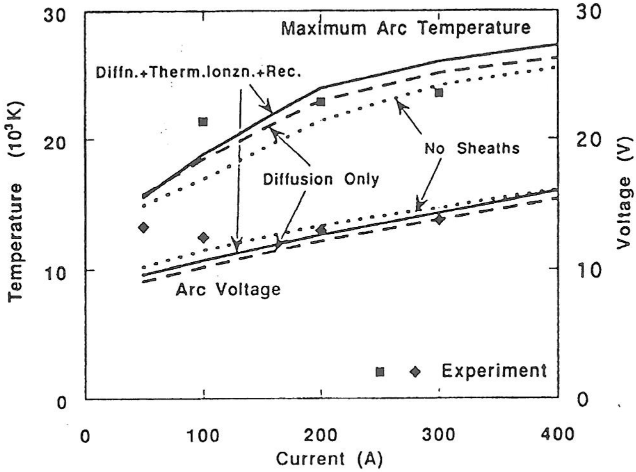

Fig.2. Predicted Maximum Arc Temperature and total Voltage as a function of Current.

Summary

A simplified unified theory of free burning arcs and their electrodes has been developed. Predictions of maximum arc temperatures and arc voltages as a function of current compare well with experimental results. Converged solutions are obtained in about 1/100 th of the computational time of an earlier method, which included an account of electron heating in the cathode sheath, but this effect can be neglected. For very approximate engineering calculations, the effect of sheaths can be neglected, provided optimum grid sizes are chosen near the electrodes.

References

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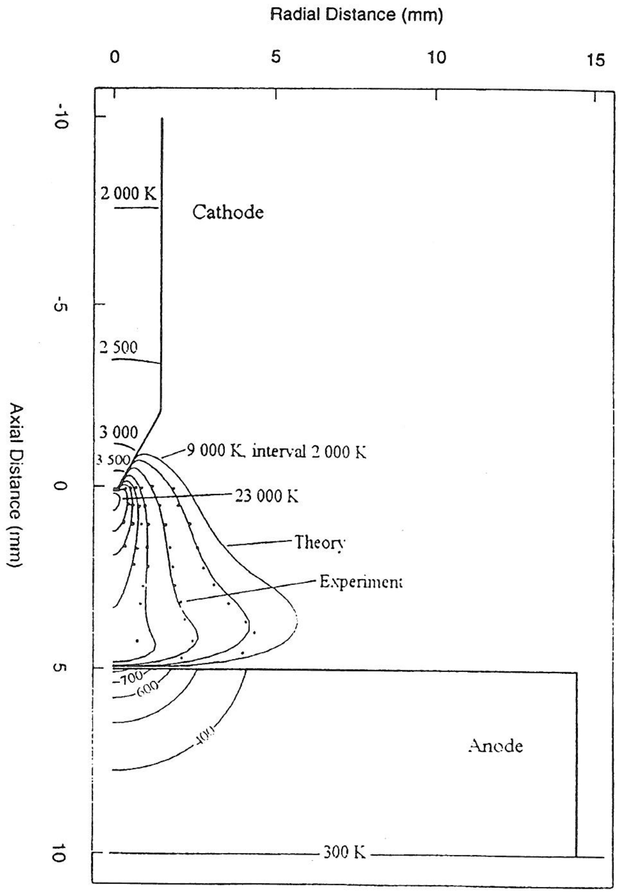

Fig. 1 The predicted temperature contours for a 200 A arc compared with experiment.

Fig. 2 shows predicted curves giving the maximum arc temperature and the total voltage as functions of current. Also shown are: (a) results with only diffusion included in the electron continuity equation at the cathode and (b) results obtained simply by taking the local electrical and thermal conductivity at cathode and anode without any modifications for the effects of sheaths. The results obtained using only diffusion or local parameters are a significant function of the mesh size, whereas the curves given by the solid lines with the full treatment for the cathode sheath are remarkably insensitive to mesh size. For very approximate engineering calculations, sheaths can be neglected if the mesh size near the electrodes is optimal; eg 0.005cm for 200A. Experimental results shown in Fig. 2 as