Time-iterative Solutions of Inviscid Supersonic d.c. Plasma Jet

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Abstract

A two-dimensional numerical code is developped in order to solve the inviscid supersonic d.c. plasma jet. Radiative losses using the net emission coefficient method are included. The conservative form of the Euler equations is solved using a Runge-Kutta explicit finite difference method. Maxwell's equations and Ohm's law are solved using a Gauss-Seidel scheme. Numerical solution of the aerodynamic and electric fields are presented for a 14.5 kW plasma jet.

Introduction

The low pressure d.c. plasma jet is a technological tool that has found many applications in engineering. Plasma spray coating benefits from the reduced pressures, high velocities and long plasma core that exists in the supersonic core of the plasma jet. The supersonic plasma jet nozzle design has also received in the recent years a lot of attention from aerospace, for its potential application as a satellite thruster. The arcjet thruster is in fact an alternative to more conventional thrusters used in the aerospace industry.

The supersonic plasma nozzle is the essential tool for these technologies and there is considerable interest in developing reliable models that would enable one to design efficiently these nozzles. A model of the flow in the nozzle of a supersonic plasma jet generator involves an intricate set of coupled partial differential equations with parabolic, elliptic and hyperbolic nature and therefore presents quite a challenging task.

Governing equations

One difficulty in analytical investigations of the d.c. nozzle flow is that the flow and electromagnetic fields change largely along the position in both the radial and axial directions [1]. It is therefore necessary to introduce a two-dimensional formulation for the flow fields in order to obtain a suitable understanding of the various phenomena in presence. In the present study, a two-dimensional numerical code is developed using a hybrid approach: the aerodynamic flow fields are assumed to be expressed by a set of modified Euler equations in which heat generation and radiative losses are included, while the electromagnetic fields are expressed by the Maxwell equations.

Governing equations for aerodynamic flow fields

The governing equations for a compressible, inviscid fluid in the absence of body force and in local thermal equilibrium [2] can be written in conservative form as follows:

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = S \qquad (1)$$

with

$$Q = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{Bmatrix} \quad F = \begin{Bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (\rho e + p)u \end{Bmatrix} \quad G = \begin{Bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (\rho e + p)v \end{Bmatrix} \quad S = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{J_x^2 + J_y^2}{\sigma} - \varepsilon_n \end{Bmatrix} \qquad (2)$$

where e is the total energy per unit volume, u and v are the x and y components of the velocity, p is the pressure and J_x and J_y are the components of the current density vector. The source term S includes Joule heating and radiative losses using the net emission coefficient method [3]. The fluid (argon) is assumed to obey the perfect-gas law. The molecular weight of the partially ionized plasma gas is evaluated with Dalton's law by solving the Saha equilibrium relation for thermally ionized plasma gases :

$$\frac{\alpha^2}{1-\alpha^2} = \left(\frac{U_{Ar}}{U_{Ar+}}\right)\left(\frac{2\pi m_e}{h^2}\right)^{3/2} \frac{\left(k_b T\right)^{5/2}}{p} e^{\left(\frac{-\varepsilon_i}{k_b T}\right)} \qquad (3)$$

where α is the ionized fraction, U_{Ar}, U_{Ar+} are the partition function, m_e the mass of the electron, ε_i is the ionization energy, h and k_b the Planck and Boltzmann constants and p the pressure. The electric conductivity σ is a critical transport property since it controls the discharge current distribution. The values are due to Murphy[4].

Governing equations for electric field

If the magnetic fields are neglected, the Maxwell's equations and Ohm's law can be respectively written for the steady-state electric field as follows:

$$\nabla \bullet \mathrm{j} = 0 \qquad \nabla \times \mathrm{E} = 0 \qquad \mathrm{j} = \sigma \mathrm{E} \tag{4}$$

where j is the current density vector, E is the electric field. The electric field can then be defined by a gradient vector of the electric potential ϕ as given by

$$\mathrm{E} = \nabla \phi \tag{5}$$

wich yields

$$\nabla \bullet (\sigma \nabla \phi) = 0 \tag{6}$$

This equation is meaningful only within the electrically conductive domain, where $\sigma \neq 0$. According to Joule's law, the ohmic heating can then be computed as:

$$\mathrm{S} = \sigma(\mathrm{E} \bullet \mathrm{E}) = \frac{J_x^2 + J_y^2}{\sigma} \tag{7}$$

Computational grid system

An elliptic grid generator [5] is used, solving a Dirichlet problem of the Laplace equation, and the physical domain is transformed onto a computational domain. Transformed equations are more complicated than the original Cartesian equations but offer several advantages [6] such as boundary surfaces treatment. The physical plane is mapped onto the computational one, and equation 1 becomes:

$$\frac{\partial \tilde{Q}}{\partial t} + \frac{\partial \tilde{F}}{\partial x} + \frac{\partial \tilde{G}}{\partial y} = \tilde{S} \tag{8}$$

with

$$\tilde{Q} = \frac{Q}{J} \quad \tilde{F} = \frac{\zeta_x F + \zeta_y G}{J} \quad \tilde{G} = \frac{\eta_x F + \eta_y G}{J} \quad \tilde{S} = \frac{S}{J} \tag{9}$$

where J is the transformation Jacobian, defined by :

$$J = \varsigma_x \eta_y - \varsigma_y \eta_x \tag{10}$$

Numerical procedure

The conservative form of the two-dimensional, time-dependent Euler equations is solved using a Runge-Kutta explicit finite difference method [7]. The convective terms are approximated by the second-order central difference expression, therefore the addition of damping terms [8] is required to prevent oscillations in the region of the flow field where strong pressure gradients exist. This damping is applied in both η and ς directional sweeps.

The electric fields are calculated in the time-steady form, separately from the aerodynamic flow field since time intervals determined for that field by CFL values are generally enough to regard the electric field in steady-state [1]. A Gauss-Seidel scheme is used to solve the electric field.

As a first step, the isentropic one-dimensional flow is solved given the temperature and pressure at the inlet. With this solution as an initial condition, the d.c. nozzle flow including heat generation and radiative losses is calculated. The electric field is then calculated and the electric potential ϕ and discharge current density vector J are determined. These calculations are repeated until both the electric and aerodynamic flow fields become steady. After each sequence, the electrical conductivity σ and degree of ionisation α are evaluated based on the known gas pressure and temperature.

The model was validated for the case of a 2-dimensional isentropic nozzle flow, based on the results of Kliegel and Quan [10]. The agreement was found to be excellent.

Results and discussion

The numerical solution of equation 8 describing the aerodynamic flow fields was made on a 60 by 40 finite difference grid, a typical run taking approximately 40,000 iterations to converge. The criterion for convergence used in this study was that the residue of density had to be lower that 10^{-7}. The CFL number, based on the average flow conditions, is 0.86 for the time step of 0.15 µs that was chosen for the computations. The electric fields, equation 6, are solved every 100 flow field iterations. In order to obtain convergence an adaptive procedure was used to enable the aerodynamic fields to adapt to the power dissipation imposed by the given voltage drop. In the results presented here the overall voltage drop is set at 27.5 volts and the resulting power dissipation was found to be 14.5 kW.

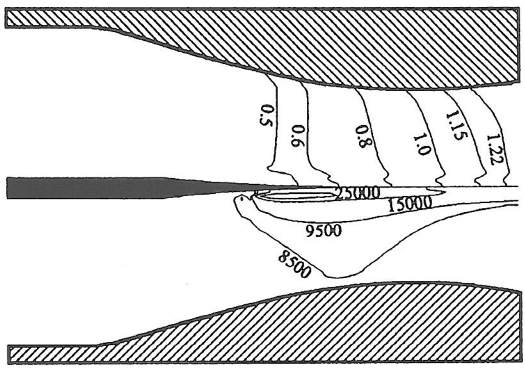

Figure 1. Isomach(top) and isotherms(bottom).

Figure 1 shows the aerodynamic and temperature fields for the plasma jet with an atmospheric inlet pressure. The dimensions of the nozzle are:

Length 50 mm .

Throat radius 9.4 mm.

Exit to throat ratio 1.096

It is seen that the exit Mach number is approximately 1.22, while the one-dimensional isentropic flow solution gives 1.36.

However caution must be given here in the interpretation of the Mach numbers presented, since we did not include ionization effect in the calculation of the speed of sound [9]. As expected, the throat Mach number is close to 1, with a slight increase toward the centerline. The exit velocity is of the order of 1200 m/s with a pressure of 0.39 torr. The maximum temperature is of the order of 25,000 K, close to the tip of the cathode. Temperature drop rapidly to 10,000 K approximately at the exit of the nozzle. Strong temperature gradients are present both axially and radially.

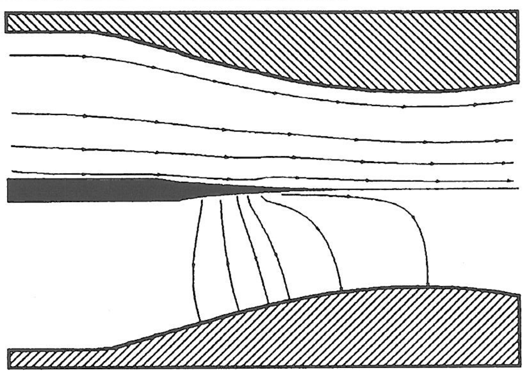

Figure 2. Stream lines(top) and electric current lines(bottom).

Figure 2 shows the stream lines of the flow and of the electric current. A slight deviation of the innermost streamline indicates an slight increase in pressure at the tip of the cathode, due to the strong ohmic dissipation and expansion of the gas associated. The bottom portion of Figure 2 shows the direction of the current from the cathode to the anode, concentrated in the plasma region of the flow.

Conclusions

A model of the inviscid supersonic d.c. plasma jet in the nozzle region has been developped. Based on the hypothesis that the flow is governed by sources (electrical, radiative and pressure terms) and by convective terms, the model has been used to predict the aerodynamic and temperature fields as well as the electric fields in the nozzle. Maximum temperatures are in the range of 25,000 K at the tip of the cathode for a power of 14.5 kW, and the Mach number at the exit is 1.22. Even at this modest Mach number it is shown that there is a significant difference between the classical 1-D isentropic solution and the model prediction. Further developments of the present model are underway, including diffusion effects and systematic studies of the geometric design of the nozzles. The effect of the geometry on the thruster performance, both in electric and in aerodynamical terms will be studied.

Acknowledgments

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