MATHEMATICAL MODEL FOR THE CALCULATION OF THE IMPEDANCE OF R.F. INDUCTIVELY COUPLED PLASMAS

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Abstract
The model used for the calculation of the plasma impedance is based on the simultaneous solution of the 2-D momentum, continuity and energy equations with the corresponding electromagnetic fields in the induction plasma, formulated in terms of the vector potential. The computation of the equivalent plasma impedance was carried out for an Ar/H_2 gas plasma generated in a 50 mm plasma confinement tube with an oscillator of frequency 3 MHz and at a plasma power of 50 kW. The results show important variations of the plasma impedance and coupling efficiency with the plasma composition .

1. Introduction

Considerable attention has been given to the mathematical modelling of the flow, temperature and concentration fields in inductively coupled radio frequency (r.f.) plasmas [1-6]. While these studies provide valuable information about the 2-D electromagnetic fields in the discharge [4], they stop short of a complete calculation of the electrical impedance of the plasma which is an important factor for the design of power supply impedance matching circuits [7,8]. The objective of the present study is to develop a global model where the flow and temperature fields in the discharge are coupled to the overall electrical characteristic of the circuit (oscillator/plasma system). Freeman and Chase [11] were the first to adopt the simple channel model, based on a close analogy with the induction heating of metals where the derived equivalent plasma impedance was based on the theory of induction heating of solid metals [9-11]. They assumed the temperature and electrical conductivity inside the plasma are uniform throughout its entire radius, This assumption is generally acceptable at low oscillator frequencies, small discharges and when coil end effects are considered to be negligible . With induction plasmas, the load is a conducting plasma gas which has substantially lower electrical conductivity than most metals and is dependant on plasma temperature [12], has a direct influence of the optimal frequency and impedance matching circuit design. In the present paper the turbulent model incorporating the 2-D formulation of the electromagnetic fields through the concept of vector potential as developed by Mostaghimi and Boulos [4], is used for the calculation of the plasma impedance and coupling efficiency. The results show important variations of the plasma impedance with plasma composition.

2. Torch geometry and basic assumptions

A schematic of the (r.f.) inductively coupled plasma torch used for this model is shown in Fig. 1. The outer tube wall is water cooled and is surrounded by a five-turn induction coil with current supplied by an external (r.f.) power supply. A summary of the torch dimensions and operating conditions are given in Table 1. The plasma gas was argon and hydrogen at atmospheric pressure. The following assumption were made; negligible displacement currents, turbulent flow and heat transfer with negligible viscous dissipation, local thermodynamic equilibrium and optically thin plasma.

Table 1. Torch dimensions and operating conditions.

R_1	2 mm	dw_1	3.5 mm
R_2	4.5 mm	P	50 kW
R_3	21 mm	f	3 MHz
R_4	25 mm	Q1(Ar)	10 slpm
R_c	33 mm	Q_2 (Ar)	30 slpm
Z_1	60 mm	Q_3 (Ar/H_2)	150 slpm
Z_2	120 mm	Z_3	250 mm
Z_p	90 mm		

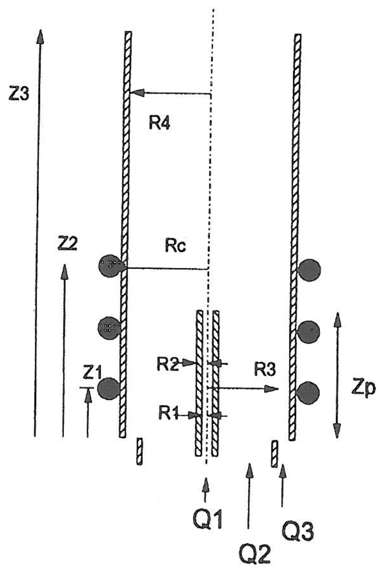


Figure 1. Torch geometry

2.1 Governing equations

The relevant equations are the continuity, momentum, energy and vector potential equations. Due to the limited publication space, these equations will not be reproduced here. Details of these equations along with the solution procedure are described in references [2,13]. The (r.f.) discharge power dissipated in the plasma load and the reactive power of the plasma system are given respectively by the following equations :

$$W_D = \int_{vol} \sigma \vec{E}^2 d(vol) \quad (1)$$

$$W_R = 2\pi f \mu_0 \int_{vol} \vec{H}^2 d(vol) \quad (2)$$

where $\vec{E}$, $\vec{H}$ the 2-D electric and magnetic fields in the plasma and σ electrical conductivity of the plasma gas. The apparent inductance (L_{eq}) of the plasma is calculated as the ratio of the (r.f.) reactive power to the square of the applied coil current. The equivalent resistance (R_{eq}) of the plasma is calculated as the ratio of the (r.f.) discharge power dissipated in the plasma to the square of the total induced current. The equivalent impedance of the plasma system viewed from the oscillator is finally derived as :

$$Z = \sqrt{(R_{eq}.m^2)^2 + (2\pi f.L_{eq})^2} \quad (3)$$

where (m) is the ratio of the total induced current to the coil current and (f)is the oscillator frequency. The coupling efficiency (η_c), which characterises the transfer of power from the coil to the discharge is calculated as :

$$\eta_c = \frac{W_D}{W_R} \quad (4)$$

This coefficient depends on the ratio of the plasma radius (R_p) to that of the coil radius (R_c) and the coupling parameter ($\sqrt{2}\,\delta/R_p$), with δ the skin depth; $\delta = 1/\sqrt{\pi\mu_0\sigma f}$

3. Results and discussion

Results are obtained for an Ar/H_2 plasma at atmospheric pressure with varying proportions of hydrogen . Two parameters are calculated, namely the equivalent plasma impedance and the coupling efficiency . Computations are carried out for a 50 mm plasma confinement tube coupled with an oscillator of 3 MHz and at a plasma power of 50 kW .

The introduction of hydrogen in the plasma gas (argon), increases the minimum power required to sustain the discharge. At a fixed power the plasma temperature drops with the increase of the hydrogen concentration, which in turn results in the decrease of the degree of ionisation and therefore the plasma electrical conductivity (Fig. 2). A decrease of the electrical conductivity results in the increase of the skin depth and more uniform distribution of electric field and energy in the plasma, as show in Fig. 3-a. The equivalent plasma inductance is smallest for 100 % argon, it increases as more hydrogen is introduced (Fig. 3-b), with the increase in the reactive discharge power and with the decrease of coupling efficiency (Fig. 4-a). This results in a corresponding increase in the impedance as viewed from the oscillator (Fig. 3-c). The coupling efficiency also increases with the increase of the ratio of critical plasma radius to that of the coil radius (Fig. 4-b).

4. Conclusion

A mathematical model for the calculation of the impedance of the (r.f.) inductively coupled plasma has been proposed . This provides information on the electrical parameters of the load of the plasma system, which is necessary for designing the total induction thermal plasma system. Further work is to be devoted to the study of the effects of the plasma power, coil geometry and oscillator frequency on the overall electrical characteristics of the plasma.

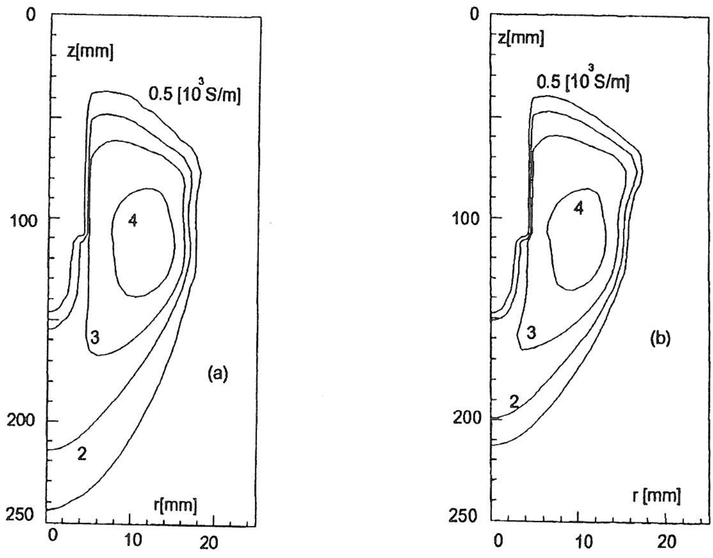


Figure 2. Isocontour of the electrical conductivity for Ar/H_2 plasmas with different concentrations of H_2 in the sheath gas. (a) 20 % vol H_2 ; (b) 30 % vol H_2

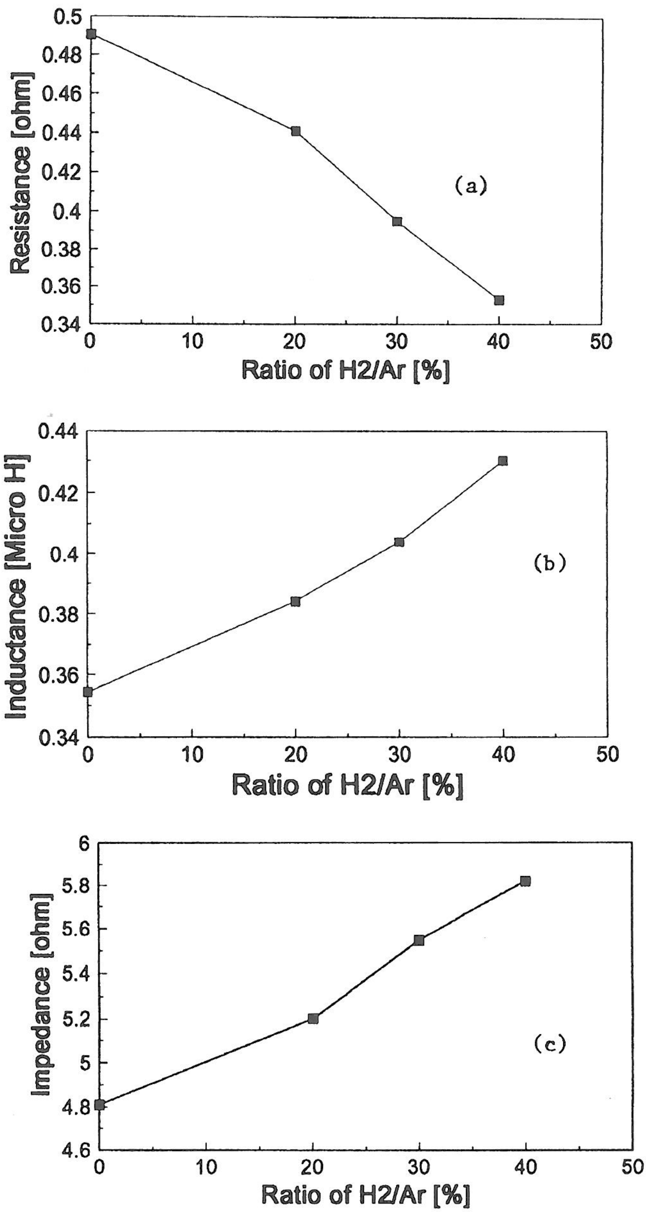
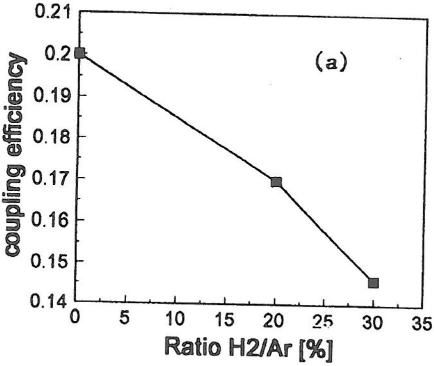


Figure 3. Equivalent plasma resistance (a); Equivalent plasma inductance (b); Impedance of the plasma system (c)



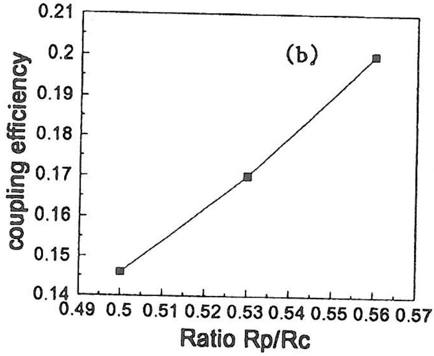


Figure 4. Coupling efficiency as function of the ratio of the $[H_2/Ar]$ and of $[R_p/R_c]$

5. Acknowledgement

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6. References

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