NUMERICAL SIMULATION OF A NON-EQUILIBRIUM PLASMA JET IN AN APPLIED MAGNETIC FIELD USING THREE-FLUID MODEL

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ABSTRACT

A three-fluid model is applied here for the numerical simulation of the axisymmetric flow and temperature fields in a non-equilibrium argon plasma jet under the mirror type magnetic field. Equations of conservation for each plasma species coupled with the generalized Ohm's law, Maxwell's equations and the equation of state are simultaneously solved taking into account variable transport properties.

INTRODUCTION

Plasma is one of the multi-functional fluids having high energy density, various working gas selectivity, and it is chemically high active and is easy to be controlled by applying the magnetic field. Therefore, applications of plasma for plasma material processing, MHD generation, plasma propulsion, plasma combustion and also environmental control etc., have been extensively developed [1]. In these industrial applications, effective utilization of its function and precise control of plasma are closely related to get higher energy conversion efficiency and process efficiency. Among a number of external control parameters, the magnetic control is effective to control precisely the plasma characteristics from both macroscopic and microscopic points cleanly and stably [2]. However, there are few released papers for detailed numerical simulation for the purpose of the magnetic control of plasma [3].

In the present study, for selective and functional control of plasma species in a non-equilibrium plasma jet by applying the mirror type magnetic field as shown in Fig.1, three-fluid model is adopted. The governing equations are derived taking into account plasma elementary processes, temperature, pressure dependent and anisotropic transport properties and interactions between plasma species. Then, we make clear by numerical simulation about the variations of the velocity and the temperature fields of electrons, ions and atoms by applying the magnetic field, and further, the energy transfer between plasma species in the magnetic field.

THEORETICAL

In order to obtain the governing equations, following assumptions are introduced as follows:

(1) Working fluid is argon gas, and plasma is regarded as ideal gas and continuum.
(2) Jet is axisymmetric subsonic laminar flow with swirl.
(3) Viscous dissipation in energy equations is neglected.

(4) Plasma is optically thin, and for elementrary process, only two-body collisional ionization and three-body recombination are considered.
(5) Self induced magnetic field is neglegibly small, and applied magnetic field is given by Biot-Savart's formula.
(6) Transport properties have temperature and pressure dependence and anisotropy.
(7) Mass difference between argon atom and argon ion is neglected.
(8) Pressure gradient current of electron ($\sigma_e \nabla p_e / e n_e$) in Ohm's law is neglected.

The governing equations for all plasma species of s kinds are summarized as follows:
conservation of mass :

$$\frac{\partial}{\partial z}(\rho_s u_s) + \frac{1}{r}\frac{\partial}{\partial r}(r \rho_s v_s) = m_s \dot{n}_s \tag{1}$$

according to the charge consevation law:

$$\dot{n}_e = \dot{n}_i = \dot{n}_n \tag{2}$$

the net production rate of electrons is given by

$$\dot{n}_e = \frac{d\, n_e}{d\, t} = k_{ion}(T_e)\; n_e n_n - k_{re}(T_e)\; n_e^2 n_i \tag{3}$$

where the ionization coefficient $k_{ion}(T_e)$ and the recombination coefficient $k_{re}(T_e)$ are yielded by

$$k_{ion}(T_e) = 8 S_1 (2\pi m_e)^{-\frac{1}{2}} (k T_e)^{\frac{3}{2}} \left(\frac{\epsilon_1}{2kT_e} + 1 \right) \exp\left(-\frac{\epsilon_1}{kT_e} \right) \tag{4}$$

$$k_{re}(T_e) = \frac{k_{ion}(T_e)}{K_{eq}(T_e)} \tag{5}$$

The equilibrium constant $K_{eq}(T_e)$ is given by Saha equation :

$$K_{eq}(T_e) = \frac{2G_i}{G_n} \left(\frac{2\pi m_e k T_e}{h_{Planck}^2} \right)^{\frac{3}{2}} \exp\left(-\frac{\epsilon_{ion}}{kT_e} \right) \tag{6}$$

conservation of momentum :

$$\begin{aligned} &\frac{\partial}{\partial z}(\rho_s u_s u_s) + \frac{1}{r}\frac{\partial}{\partial r}(r \rho_s u_s v_s) \\ &\quad = -\frac{\partial p_s}{\partial z} - \frac{2}{3}\frac{\partial}{\partial z}\left[(\eta_s)_z \left\{ \frac{\partial u_s}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r v_s) \right\} \right] + 2\frac{\partial}{\partial z}\left((\eta_s)_z \frac{\partial u_s}{\partial z} \right) \\ &\quad + \frac{1}{r}\frac{\partial}{\partial r}\left(r (\eta_s)_r \frac{\partial v_s}{\partial z} \right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r (\eta_s)_r \frac{\partial u_s}{\partial r} \right) + \frac{\partial}{\partial z}(\rho_s U_s U_s) \\ &\quad + \frac{1}{r}\frac{\partial}{\partial r}(r \rho_s V_s U_s) + m_s \dot{n}_s u_s - M_{sz} + e_s n_s (E_z' - W_s B_r) \end{aligned} \tag{7}$$

$$
\begin{aligned}
&\frac{\partial}{\partial z}(\rho_s v_s u_s) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho_s v_s v_s) \\
&\quad = -\frac{\partial p_s}{\partial r} - \frac{2}{3}\frac{\partial}{\partial r}\left[(\eta_s)_r\left\{\frac{\partial u_s}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r v_s)\right\}\right] + \frac{\partial}{\partial z}\left((\eta_s)_z\frac{\partial v_s}{\partial z}\right) \\
&\qquad + \frac{\partial}{\partial z}\left((\eta_s)_z\frac{\partial u_s}{\partial r}\right) + \frac{2}{r}\frac{\partial}{\partial r}\left(r(\eta_s)_r\frac{\partial v_s}{\partial r}\right) - 2(\eta_s)_r\frac{v_s}{r^2} \\
&\qquad + \frac{\partial}{\partial z}(\rho_s V_s U_s) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho_s V_s V_s) - \rho_s\frac{W_s^2 - w_s^2}{r} + m_s\dot{n}_s v_s \\
&\qquad - M_{sr} + e_s n_s(E_r' + W_s B_z)
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial z}(\rho_s w_s u_s) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho_s w_s v_s) \\
&\quad = \frac{\partial}{\partial z}\left((\eta_s)_z\frac{\partial w_s}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r(\eta_s)_r\frac{\partial w_s}{\partial r}\right) - \frac{w_s}{r^2}\frac{\partial}{\partial r}\left(r(\eta_s)_r\right) \\
&\qquad + \frac{\partial}{\partial z}(\rho_s W_s U_s) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho_s W_s V_s) + \rho_s\frac{V_s W_s - v_s w_s}{r} \\
&\qquad + m_s\dot{n}_s w_s - M_{s\theta} + e_s n_s(E_\theta' + U_s B_r - V_s B_z)
\end{aligned}
\tag{9}
$$

conservation of energy :

$$
\begin{aligned}
&\frac{\partial}{\partial z}\left(\frac{5}{2}n_s u_s k T_s\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{5}{2}r n_s v_s k T_s\right) \\
&\quad = \frac{\partial}{\partial z}\left((\lambda_s)_z\frac{\partial T_s}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r(\lambda_s)_r\frac{\partial T_s}{\partial r}\right) + \left(u\frac{\partial p_s}{\partial z} + v\frac{\partial p_s}{\partial r}\right) \\
&\qquad + \frac{\partial}{\partial z}(\rho_s H_s U_s) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho_s H_s V_s) + \frac{5}{2}\dot{n}_s k T_s - Ex_s \\
&\qquad + (J_{sz}E_z' + J_{sr}E_r' + J_{s\theta}E_\theta')\underline{- \dot{R} - \epsilon_{ion}\dot{n}_e}
\end{aligned}
\tag{10}
$$

where the underlined terms in equation (10) is added only for electrons.
Ohm's law :

$$
E_z' = \left\{1 + s\left(\frac{B_r}{B}\right)^2\right\}\frac{J_z}{\sigma_e} - s\left(\frac{B_z}{B}\right)\left(\frac{B_r}{B}\right)\frac{J_r}{\sigma_e} - \beta_e\left(\frac{B_r}{B}\right)\frac{J_\theta}{\sigma_e} \tag{11}
$$

$$
E_r' = -s\left(\frac{B_z}{B}\right)\left(\frac{B_r}{B}\right)\frac{J_z}{\sigma_e} + \left\{1 + s\left(\frac{B_z}{B}\right)^2\right\}\frac{J_r}{\sigma_e} + \beta_e\left(\frac{B_z}{B}\right)\frac{J_\theta}{\sigma_e} \tag{12}
$$

$$
E_\theta' = \beta_e\left(\frac{B_r}{B}\right)\frac{J_z}{\sigma_e} - \beta_e\left(\frac{B_z}{B}\right)\frac{J_r}{\sigma_e} + (1+s)\frac{J_\theta}{\sigma_e} \tag{13}
$$

J_z, J_r, J_θ are components of current density in z, r, θ directions and they are given as following equations :

$$J_z = J_{iz} + J_{ez} = e(n_i U_i - n_e U_e) \tag{14}$$

$$J_r = J_{ir} + J_{er} = e(n_i V_i - n_e V_e) \tag{15}$$

$$J_\theta = J_{i\theta} + J_{e\theta} = e(n_i W_i - n_e W_e) \tag{16}$$

equation of state :

$$p_s = n_s k T_s \tag{17}$$

The momentum transfer M_s and the energy transfer Ex_s by collision between s species particle and r species particle are given as following equations respectively.

$$M_s = \sum_r n_s \bar{\nu}_{sr} m_{sr} (u_s - u_r) \tag{18}$$

$$Ex_s = \sum_r \frac{2m_{sr}}{m_s + m_r} \frac{3}{2} k (T_s - T_r) \bar{\nu}_{sr} n_s \tag{19}$$

Here, $\bar{\nu}_{sr}, m_{sr}$ are the elastic collision frequency and the conversion mass respectively. The viscosity η_s and the thermal conductivity λ_s of each plasma species are obtained by following equations using the mean free path length ℓ_s and the thermal velocity c_s.

$$\eta_s = \frac{5\pi}{32} m_s n_s \ell_s c_s \tag{20}$$

$$\lambda_s = \frac{75\pi}{128} n_s k \ell_s c_s \tag{21}$$

At the arbitrary ionization degree, the electrical conductivity σ_e is given using the electrical conductivity of fully ionized plasma σ_c and that of partially ionized plasma σ_p.

$$\sigma_e = \left[\frac{1}{\sigma_c} + \frac{1}{\sigma_p} \right]^{-1} \tag{22}$$

Finally, the anisotropic transport properties are given using Hall parameter β_s and the ion slip coefficient s.

$$\begin{pmatrix} \eta_s \\ \lambda_s \\ \sigma_e \end{pmatrix}_z = \left(\cos\phi - \frac{1+s}{(1+s)^2 + \beta_s^2} \sin\phi \right) \begin{pmatrix} \eta_s \\ \lambda_s \\ \sigma_e \end{pmatrix} \tag{23}$$

$$\begin{pmatrix} \eta_s \\ \lambda_s \\ \sigma_e \end{pmatrix}_r = \left(\sin\phi + \frac{1+s}{(1+s)^2 + \beta_s^2} \cos\phi \right) \begin{pmatrix} \eta_s \\ \lambda_s \\ \sigma_e \end{pmatrix} \tag{24}$$

where ϕ means the gradient of the magnetic lines of force which shows counter clockwise from z axis.

Above the governing equations from eq.(1) to eq.(17) are to determine the number density, the velocity and the temperature fields for plasma species and the induction electric field. As the boundary conditions shown in Fig.1, there are non-slip condition and adiabatic condition at the wall BC and CD. There are symmetrical condition at the centerline AE. At the outflow boundary DE, the mass flow rate is conserved and the radial component of the velocity, and further the second order gradients of the swirl velocity and the temperature are zero values.

NUMERICAL RESULTS AND DISCUSSION

The outflowing conditions at the nozzle exit AB are constant total enthalpy, constant mass flow rate. As ideal conditions every plasma species are assumed to have the equal velocity ($u_n = u_i = u_e$) and the equal temperature ($T_n = T_i = T_e$). The axial velocity and the temperature distributions are one dimensional top hat at the nozzle exit. To utilize the axisymmetric magnetic effect, the swirl velocity of $w_s/u_0 = 0.1r/r_0$ is given at the nozzle exit. The calculation conditions of the plasma jet and at the nozzle exit are listed in Table 1. For governing equations, the flow and the temperature fields are calculated using SIMPLE method, and the magnetic field is calculated by the control volume method.

Figures 2(a) to (f) show the isocontours of the temperature and the velocity of each plasma species for $B_{\max} = 0,\ 1.0$[T]. In the region from the outer jet fringe to near wall, where collision between plasma species is poor, then it is shown the thermal non-equilibrium as $T_e > T_i \approx T_n$. And the temperature of each plasma species increases in the region from jet fringe to near wall by applying the magnetic field. The electron temperature increase in this region is considered to cause the increase in the degree of ionization. And the electron velocity is 1.2 times faster than the heavy particle velocity, and it is changed in near central region by applying the magnetic field. This is considered why the mass and the viscosity of electron is considerably smaller than those of heavy particle. And at just after the nozzle exit, the ion velocity is rather faster than the atom velocity due to the induced electric field, but in the downstream they become almost same and both are scarcely influenced by the magnetic field.

Figures 3(a) to (e) show the variation of the electron velocity by the different applied magnetic field intensity $B_{max} = 0.0,\ 0.25,\ 0.5,\ 0.75$ and 1.0 T. With the increase of magnetic field intensity, the electron velocity decreases sharply immediately after the blowout from the nozzle exit and later increases once along the axial direction, and then decreases again. And by even the small magnetic field intensity, the electron velocity at plasma core shows remarkable change.

Figure 4 shows the variation of the energy transfer between plasma species on eq.(19) by different magnetic field intensity. The energy transfer between the heavy particles is little influenced by the magnetic field intensity, but the energy transfer between the electrons and heavy particles increases sharply at first, and later it increases gradually with the magnetic field intensity. This is because that under even small magnetic field intensity, the electrons are constrained at near central axis, then the number density and also collision frequency increase.

CONCLUDING REMARKS

In this paper, a theree fluid model has been applied for the numerical simulation of a non-equilibrium argon plasma jet with various magnetic fields ranging from 0 to 1.0 T. The calculation indicated that the electron temperatures are higher than the heavy particle temperatures in the domain from the jet fringe to the wall, and the velocities of electrons are strongly influenced by the magnetic field. The effects of the magnetic field on the energy transfer between the plasma species have been discussed.

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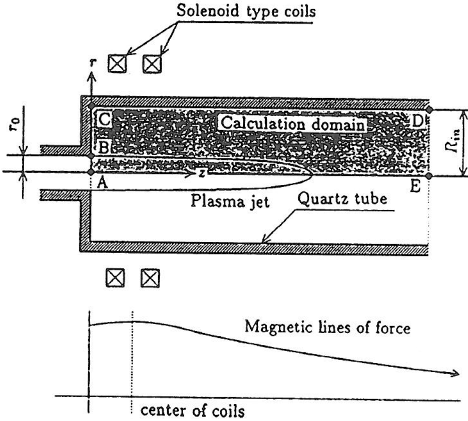

Fig.1 Nonequilibrium plasma jet in the applied magnetic field

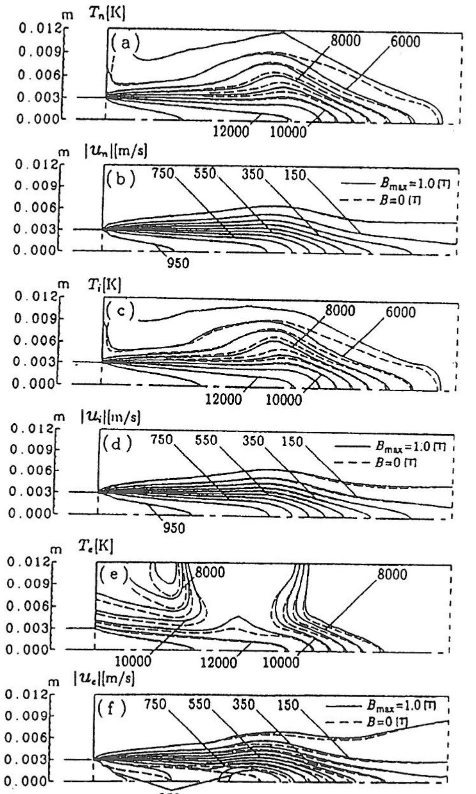

Fig.2 Isocontours of temperature and velocity for plasma species

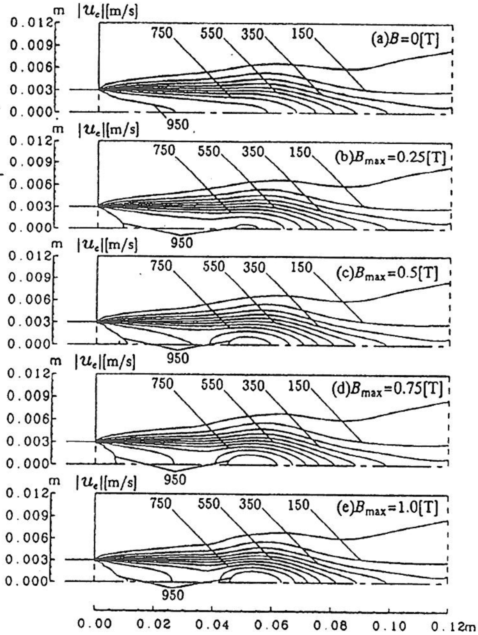

Fig.3 Variation of electron velocity by the magnetic field

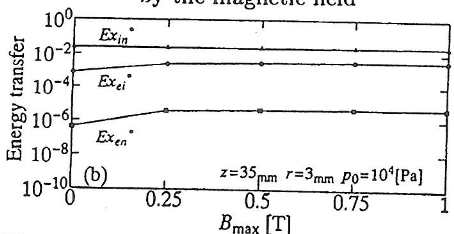

Fig.4 Variation of energy transfer between plasma species by the magnetic field

Table 1 Calculation conditions

Working gas	Argon
Gas constant of argon	R = 208.1 J/(kg·K)
Specific heat (p = 1.013 × 10^4Pa, T_0 = 13495.3 K)	C_{p0} = 10.16 kJ/(kg·K)
Nozzle radius	r_0 = 3.0 × 10^{-3} m
Tube radius	r_{in} = 12.0 × 10^{-3} m
Tube length	L_0 = 120.0 × 10^{-3} m
Total enthalpy at Nozzle exit	h_0 = 1.2 × 10^8 J/kg
Mass flow rate	$\dot{m}$ = 1.0 × 10^{-4} kg/s
Input power	P_{input} = 12.0 kW
Pressure at nozzle exit	p_0 = 1.013 × 10^4 Pa
Maximum magnetic flux density	B_{max} = 0.0, 0.25, 0.5, 0.75, 1.0 T