ON THE ADEQUACY OF APPROXIMATION METHODS FOR THE CURRENT-VOLTAGE CHARACTERISTICS OF ARC PLASMA TORCHES

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ABSTRACT

For current-voltage characteristics (CVC) of vortex plasma toches, the net r.m.s. error is estimated by means of experimental repetition. The part of r.m.s. deviation from regression which is caused by approximation non adequacy is separated. The power expression is tested for cvc approximation and shown to be quite acceptable for this purpose.

INTRODUCTION

The theoretical determination of arc plasma torches characteristics runs into substantial difficulties due to the phenomena complexity. The solution is obtainable only for rough assumption of arc symmetry, stability, simple discharge chamber geometry, etc. However, rather unstable arcs are usually applied for different purposes. Experimental data approximations, both in the form of simple empirical expressions or by means of generalized relationships, are widely used at present. However the problem of approximate expressions adequacy at a given set of variables arises in any case.

In order to reduce the error at the expense of approximate expressions nonadequancy, rather complicated relationships are sometimes used for arc characteristics. Such formulae are very unconvenient for experimental data processing. Therefore, the simple power expressions are desirable to be used for this purpose. They enable the application of linear regression for statistic data processing by taking the logarithms of them. The adequacy may be estimated by the comparison of Fisher variances ratio with the table values [1]. The ratio of regression and total scattering variances is usually applied for such evaluations. The convenience of this ratio is that any data file can be analyzed. But it makes no distinction between random scatter and approximate expression deviation from the real law.

The random scattering can be separated only in the case of special experiment, when every point is repeated many times. These repetitions enable to calculate the sum of squared random scatter. Then, the sum of squared non adequancy deviation can be obtained as the difference of total and random squared departures. The same procedure is applied to degrees of freedom and the variances are obtained as the ratios of corresponding sums of squared deviations and degrees of freedom. An approximation satisfies the law when the ratio of non adequancy and random variances is less then the corresponding Fisher ratio table values.

Such a rigorous adequacy estimation is unusual for works dealing with electric arc characteristics due to the necessity of experimental points repetition. In order

to fill this gap and to find some evidence of power approximations applicability to arc plasma torchs characteristics, we have carried out special experiments. Some results of this attempt are considered below.

EXPERIMENTAL

The experiments were made with d.c. arc vortex plasma torch incorporating a rod zirconium cathode and a copper tube anode. The anode diameter amounted to 9.2 mm, and its length was adjusted to the arc length change over the range of plasma torch parameters variation. Arc current changed from 38 to 146 A, air flow-rate was set to 8 values (1.26, 1.64, 2.05, 2.45, 2.94, 3.35, 3.38 and 4.27 g/s). The current was step-wise controlled with 6 steps at every flow-rate magnitude.

Every experimental point was repeated six times, but we did not manage to fix the current in some cases. That resulted in an increase of the number of points at the expense of the number of repetitions. The total number of experiments accounted for $8 \times 6 \times 6 = 288$.

The experiments show weak voltage dependence on current and rather substantial influence of gas flow rate. The rise of gas flow-rate causes the voltage enhancement and entails the displacement of stable operation domain to the smaller currents.

To approximate plasma torch CVC, a power expression of the form $U = cI^\alpha$ is used for every given gas flow-rate and for the whole data file when G is ignored. And also, for the whole file $U = cG^\beta$ relationship is applied at the neglect of I, and $U = cI^\alpha G^\beta$ form is taken when no regression is neglected. The random scatter caused by repetition at every experimental point was calculated as the sum of squared deviations from the mean voltage value at this point. The number of random degrees of freedom at the i point equals $n-1$ (n - the number of repetitions). Then, by summing of squared deviations and degrees of freedom for all points and by subsequent division one can find the random variances for any given gas flow-rate or for the whole file. The non adequancy variances were calculated as mentioned above.

RESULTS AND DISCUSSION

Table 1 lists the regression parameters obtained for plasma torch CVC approximation. The three upper lines belong to particular gas flow-rates. They are followed by the whole file lines: G neglect, I absence and full regression, respectively. Table 1 shows that relative r.m.s. deviations from regression at given gas flow-rate are rather small amounting to some parts percent. The error of $U = cI^\alpha G^\beta$ approximation is of the same order. But one of the variables neglect brings about essential rise of scattering especially when gas flow-rate effect is ignored. $U - G$ correlation coefficients are close to unity, but $U - I$ coefficients are high only at particular G. Student quantiles are higher then table values in all cases ($t_{tabl} = 3.291$). These results confirm the strong effect of gas flow-rate on the arc voltage and its rather weak dependence on arc current.

The values of Fisher variance ratio F demonstrate the acceptability of power approximations for arc CVC even in the case of gas flow-rate effect neglect. But

Table .1: Regression parameters for CVC d.c. arc vortex plasma torch with rod zirconium cathode and tube copper anode

Air flow-rate $G, g/s$	Fator C	Exponent α	Exponent β	deviation $\sigma \times 10^3$	Correlation coefficients			Student quantiles		Fisher variances ratio for regression F_{reg}
					$U-I$	$U-G$	$I-G$	t_α	t_β	
4.27	871	-0.20	-	6.3	-0.984	-	-	-32	-	1018
2.45	676	-0.19	-	3.1	-0.991	-	-	-44	-	1958
1.27	617	-0.23	-	5.6	-0.978	-	-	-27	-	738
All points $U = cI^\alpha$	1200	-0.31	-	57.3	-0.602	-	-	-13	-	167
All points $U = cG^\beta$	2818	-	0.38	28.6	-	0.916	-	-	39	1522
All points $U = cI^\alpha G^\beta$	5495	0.20	0.34	8.6	-0.607	0.918	-0.262	-54	111	9829

more detail assessment of approximation fitness can be provided by evaluation of non adequacy to random variances ratios. These data are shown in Table 2.

Three parameters are listed upright in deviation columns for every regime: sum of squared deviations SS, degrees of freedom n and variance S^2. The ratio of non adequacy and random variances equals to F_{na}, which demonstrates the fitness of approximation.

Comparison of obtained non adequacy Fisher ratios F_{na} with table values F_{tabl} shows that power approximation is quite good. In most cases, F_{na} is less than F_{tabl}. But in some cases $F_{na} > F_{tabl}$. In the case of two-argument regression $V = cI^\alpha G^\beta$, F_{na} also exceeds the table values ($F_{na} = 13.7 > F_{tabl}$), and F_{na} is especially high when regression neglects one of the variables. Hence, power expressions can't be considered as ideal CVC approximations in any possible case. But the last column of table 2 shows that power approximation misfit isn't very bad: the ratios F_{na}/F_{reg} are less then one percent in most cases.

CONCLUSIONS

In this paper we considered the problem of the proper approximation relationship for the physical modelling of electric arcs characteristics. The validation has shown that the power expression of the form $V = cI^\alpha$ provides rather effective approximation. Correlation coefficients between function and dominant variables, Student quantiles, Fisher ratios for regression have appeared to be high ($K = 0.98$, $t_\alpha = 27 - 44$, $F_{reg} = 700 - 2000$). For the whole data file, the approximation $V = I^\alpha G^\beta$ demonstrates strong voltage dependence on flow-rate ($K_{V-G} = 0.92$, $t_\beta = 111$) and weaker one on current ($K_{V-I} = 0.61$, $t_\alpha = -54$, $F_r = 9830$). Comparisons of the adequacy

Table .2: Analysis of power expressions fitness for CVC approximation of d.c. arc vortex plasma torch

Gas flow-rate G (g/s)	Total deviation $SS \times 10^3$ / N / $S^2 \times 10^5$	Random scattering $SS \times 10^3$ / N / $S^2 \times 10^5$	Nonadequacy deviation $SS \times 10^3$ / N / $S^2 \times 10^5$	Nonadequacy Fisher variances ratio F_{na}	Table values of Fisher ratio F_{tabl} 5%	1%	Regression Fisher variances ratio F_{reg}	$F_{na}/F_{reg} \times 100\%$
1	2	3	4	5	6	7	8	9
4.27	1.334 / 34 / 3.925	0.145 / 21 / 0.688	1.190 / 13 / 9.155	13.3	2.23	3.12	1018	1.30
3.83	1.161 / 34 / 3.416	0.335 / 24 / 1.395	0.827 / 10 / 8.267	5.93	2.26	3.17	981	0.60
2.94	0.447 / 34 / 1.314	0.304 / 25 / 1.215	0.143 / 9 / 1.589	1.31	2.28	3.21	1579	0.08
2.57	1.413 / 34 / 4.210	0.176 / 24 / 0.471	1.314 / 9 / 14.600	31.0	2.28	3.21	709	4.40
2.45	0.333 / 34 / 0.980	0.206 / 26 / 0.791	0.128 / 8 / 1.594	2.01	2.32	3.29	1958	0.10
2.05	0.809 / 34 / 2.379	0.509 / 25 / 2.037	0.300 / 9 / 3.330	1.63	2.28	3.21	1046	0.16
1.64	0.796 / 34 / 2.341	0.472 / 22 / 2.144	0.324 / 12 / 2.360	1.68	2.23	3.12	1114	0.15
1.27	1.047 / 34 / 3.078	0.628 / 24 / 2.616	0.419 / 10 / 4.189	1.60	2.26	3.17	738	0.22
All points $U = cI^\alpha$	937.8 / 286 / 327.9	2.744 / 192 / 1.445	935.0 / 94 / 994.7	688	1.32	1.48	167	412
All points $U = cG^\beta$	234.7 / 286 / 82.05	2.744 / 192 / 1.445	935.0 / 94 / 24.64	171	1.32	1.48	1552	11
All points $V = cI^\alpha G^\beta$	21.20 / 285 / 7.440	2.774 / 192 / 1.445	18.43 / 93 / 19.82	13.7	1.32	1.48	9829	0.01

Fisher ratio with the table values have shown that in most cases of particular gas flow-rates the misfit does not exceed the limits of random scattering, and when this occurs, the nonadequacy is small. The ratio of adequacy to regression Fisher numbers has ocurred to be less then 4.5% in all cases. Power expressions may be considered acceptable for CVC approximation owing to their simplicity and ability to statistic processing. They demonstrate rather good fitness to real laws.

ACKNOWLEDGEMENTS

REFERENCES

[1]   N.R. Draper, H. Smith, Applied Regression Analysis, John Willey & Sons Inc., New York - London - Sydney, 1966.