CALCULATION OF RADIATIVE HEAT TRANSFER IN LOW-TEMPERATURE PLASMAS

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I. INTRODUCTION

In low-temparature plasmas (T = 10000 ... 100000 K) a main part of heat exchange is determined by radiation. Radiant transfer problems are as a rule solved together with the equations of gas dynamics. To solve the equations of energy and gas dynamics it is necessary to calculate the radiation fluxes and their divergencies in any point of space taking into consideration variable fields of temperature. Direct integration of spectral radiative transfer properties over the entire optical spectrum of complex multicomponent plasmas during solving of gas-dynamic problems is practically impossible.

Here the time needed for calculating of the radiation field when all the required radiation processes have been correctly incorporated may exceed by an order of magnitude the time needed for calculating the gas dynamics equations. Engineering methods based on empirical data are valid only over narrow ranges of variables and may yield unpredicatable errors. Model approaches, based on assumptions of a special form of the spectrum (stepwise models), of small or large optical thickness, of separation of the frequency and temperature dependencies, and so forth are excessively rough versions of the real spectrum.

In [1] was proposed and developed the method of calculations of radiative fields in spectra of any complexibility - the asymptotic integral method of Partial Characteristics (PCh). The meaning of the method is based on the fact that in mass calculations many parts associated with radiation transport are repeated to a significant extent.

The PCh method makes it possible to separate the gas dynamic and radiative parts of the problem. First one calculates all the necessary, spectrally integral parameters of radiative heat transfer. The form of these parameters, which are Partial Characterisics consisting of special functionals of the field of thermodynamic parameters - pressure and temperature, is determined.

The integration with respect to frequency is performed over the entire real spectrum, including an arbitrary number of spectral lines. No assumptions are made concerning the spectral line shapes. Nonanalytic dependence on frequency and other parameters, asymmetry, shift, multiplet splitting, and the effect of statistical microfields can be incorporated in the calculations. After the Partial Characteristics have been calculated, it is possible to solve problems of radiation gas dynamics.

The Method of PCh is asymptotically precise, i.e. if the quantity of the Reference Temperatures is large enough the error can be insignificant. This method was tested for two Reference Temperatures in [1] and independently in [2-4]. As a result the agreement to within 20...40% for divergence was obtained with exact calculations, but with a reduction of computation time of about four orders of magnitude.

This report presents new results of application of the Method of Partial Characteristics with Several Reference Temperatures. The convergence of calculations to precise results is shown.

II. CALCULATION OF RADIATION TRANSFER

The base of the method is the following. For a given gas for a number of model temperature (and pressure if P = var) profiles one calculates spectrum-integral Partial Characterisics . The PCh contain a source function and the optical density of the absorption path. As the model profiles one can use splines, which are piecewise-continuous smooth functions. The partial characteristics will be functionals of the splines.

Using the precalculated Partial Characteristics the radiation field is evaluated by simple integration. The parameters of splines needed for the selection of Partial Characteristics are found by approximating real temperature or pressure distributions by model splines.

When the order of approximation is sufficiently high, the method yields asymptotically exact results.

The method of PCh makes it possible to calculate the divergence of radiation flux in the volume of any configuration. In the point x the divergence can be evaluated by integration over space:

$$\mathrm{div}S(x) = \int \mathrm{div}I(x)\ \Omega\ d\Omega \qquad (1)$$

where (see Fig. 1):

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x   Som--→  Π                          ←--ΔSim   s              L
⋮---------⊥-------------------------------⋮-------------┤
dx        dΠ        Fig. 1                    ds
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$$\mathrm{div}I(x) = Som(x,L) - \int_x^L \Delta Sim(s, x)\ ds \qquad (2)$$

$$Som(x,L) = \int_0^\infty B\nu_0(Tx)\ K\nu(Tx)\ \exp\left(-\int_x^L K\nu(\Pi)d\Pi\right)\ d\nu \qquad (3)$$

$$\Delta Sim(s,x) = \int_0^\infty (B\nu_0(Ts) - B\nu_0(Tx))K\nu(Ts)K\nu(Tx)\exp\left(-\int_x^s K\nu(\Pi)d\Pi\right)\ d\nu \quad (4)$$

The internal integrals in Eqs. (3) and (4) are taken over the corresponding splines (x--->L), L is the border (limit) of the beam.

III. APPROXIMATION OF TEMPERATURE DISTRIBUTION

Approximation of the working temperature profile by a spline is shown in Fig.2. The reference temperatures for the spline can be selected by using points of the temperature profile.

In Fig.2 the approximation of the absorption path for three elements of beem s: 1, 2 and 3 is represented. The approximation of the path (s -> x) by one linear spline (Ts -> Tx) (dotted sparse lines) is rough enough and can bring to an error up to 40% in divergence. The approximation by three reference temperatures: Ts, T2, Tx (dotted tight lines) provides an error less then 20%.

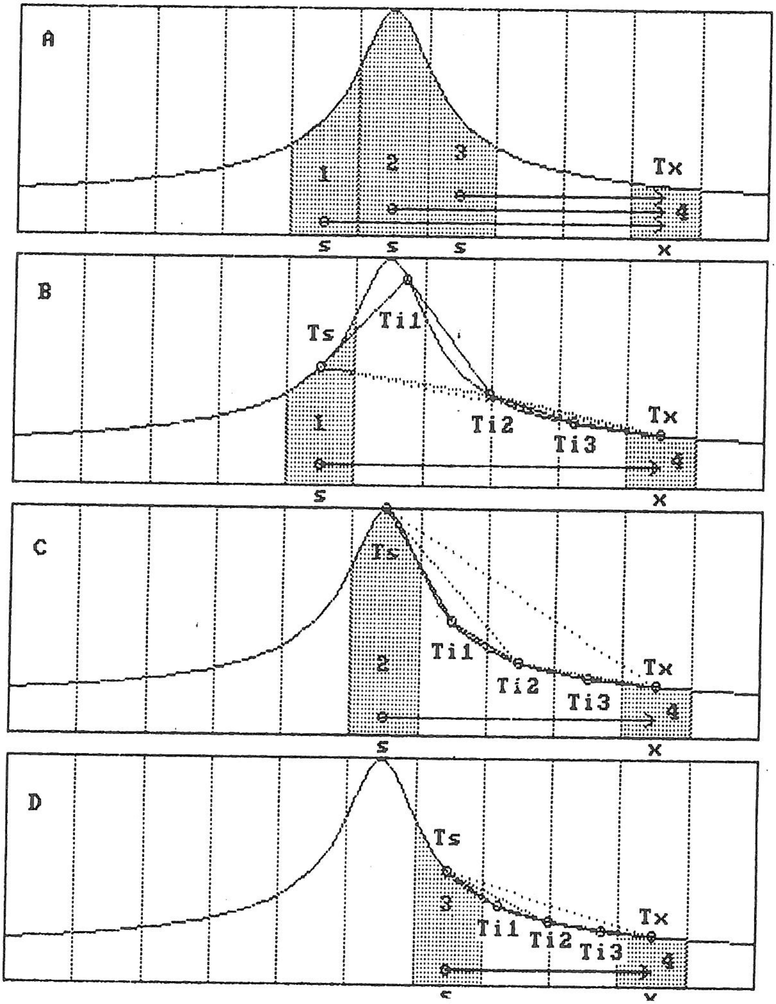

Fig.2

The approximation by five reference temperatures: Ts, T1, T2, T3, Tx (solid lines) provides practically precise results of calculations.

Note that the Partial Characteristics depend on their parameters in various ways. The functional Som depends most on the source temperature Tx, ΔSim depends most on the temperatures Ts and Tx. To facilitate interpolations it is best to enter the computer memory nondimensional quantities in the form:

$$\langle Som \rangle = Som(Tx, Ti1, Ti2, Ti3, Ts, L) / Som(Tx, 0) \qquad (5)$$

$$\langle \Delta Sim \rangle = \Delta Sim(Ts, Ti1, Ti2, Ti3, Tx, L) / \Delta Sim(Ts, Tx, 0) \qquad (6)$$

The weaker dependence of ⟨Som⟩ and ⟨ΔSim⟩ on all the temperatures Ts, Ti1, Ti2, Ti3, Tx makes it possible to tabulate ⟨Som⟩ and ⟨ΔSim⟩ with a large step on temperatures, which is important in reducing the memory volume required. At the same time the quantities Som(Tx, 0) and ΔSim(Ts, Tx, 0) which are functions only of Tx and Ts, Tx should be tabulated in a detailed manner, which allows improving the accuracy of the final calculations.

So for plasmas in interval T = 5000 ... 30000K it is enough five temperature intervals (ΔT=5000K). In this case the main arrays ⟨Som⟩ and ⟨ΔSim⟩ in spite of the large dimension (six) can have not so much volume (for example N = 5*5*5*5*5*7 = 21875).

Note that dependence of x also can be relaxed by multiplicative factor x^m, where m is of order unit.

There is the only difference in calculating arrays Som and Sim for two reference temperatures and for many reference temperatures - it is the estimation of optical density. When one has calculated the array of Kν(i) and array of ΣKν(i) for all the temperatures (for current frequency) the estimation of optical density for two reference temperatures Ts and Tx is produced as

$$\tau = \frac{\Sigma K\nu(is) - \Sigma K\nu(ix)}{(is - ix)} * \Delta x \qquad (7)$$

where indexes is and ix correspond to the temperatures Ts and Tx. In the case of many references temperatures (for example five)

the estimation of optical density is:

$$\tau = \frac{\Sigma K\nu(is) - \Sigma K\nu(i1)}{(is - i1)} * \Delta x/4 + \frac{\Sigma K\nu(i2) - \Sigma K\nu(i1)}{(i2 - i1)} * \Delta x/4 + \qquad (8)$$

$$+ \frac{\Sigma K\nu(i3) - \Sigma K\nu(i2)}{(i3 - i2)} * \Delta x/4 + \frac{\Sigma K\nu(ix) - \Sigma K\nu(i3)}{(ix - i3)} * \Delta x/4$$

IV. EVALUATION OF RADIATIVE FIELDS

This method was checked with a large number of calculations of the radiation field. The results of exact calcuolations involving direct integration of Eq. (1) with respect to frequency were compared with results of calculations by method of Partial Characteristics.

Usually, in checking numerical methods of integration of radiation characteristics with respect to frequency one uses isothermal volumes or volumes with a linear temperature distribution. Such a check is not suitable for the method of Partial Characteristics. This method always yields exact results, not only for homogeneous volumes but also for volumes with any linear temperature and pressure distributions. Hence, calculations performed for such volumes are useless for checking the method, but only for additional checking of the accuracy of inerpolation, sampling, integration, etc. In order to check the accuracy of the PCh method it is necessary to select significantly nonlinear distributions of the parameters. These distributions were represented by temperature profiles close to those encountered in practice, such as the profiles that occur in the investigations of electric arcs.

The divergence of radiation flux was calculated in plasma composed of 20% of O, 60% of Ar, 10% of C and 10% of Si. The schematic plot of absorption coefficient of the plasma is represented in Fig.3. Results of checking of the method of PCh are presented in Fig.4-6. The calculations in Fig.4 are performed for linear profile of temperature, which is represented also in

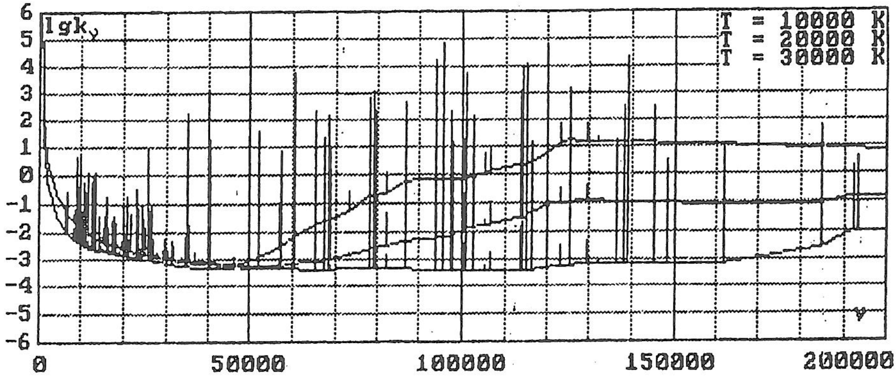

Fig.3 Absorption coefficient of Mixture O+Ar+C+Si

Fig.4. The results of calculations, obtained by direct integration via frequency (solid line) are coincided completely with calculations, obtained by the method of PCh with three parameters Ts, Tx, x (triangles). On any linear temperature profile the method of PCh gives the exact results, so this Figure serves only for the illustration, that calculation and interpolation accuracy (steps on $\Delta\nu$, Δx, ΔT) is sufficient.

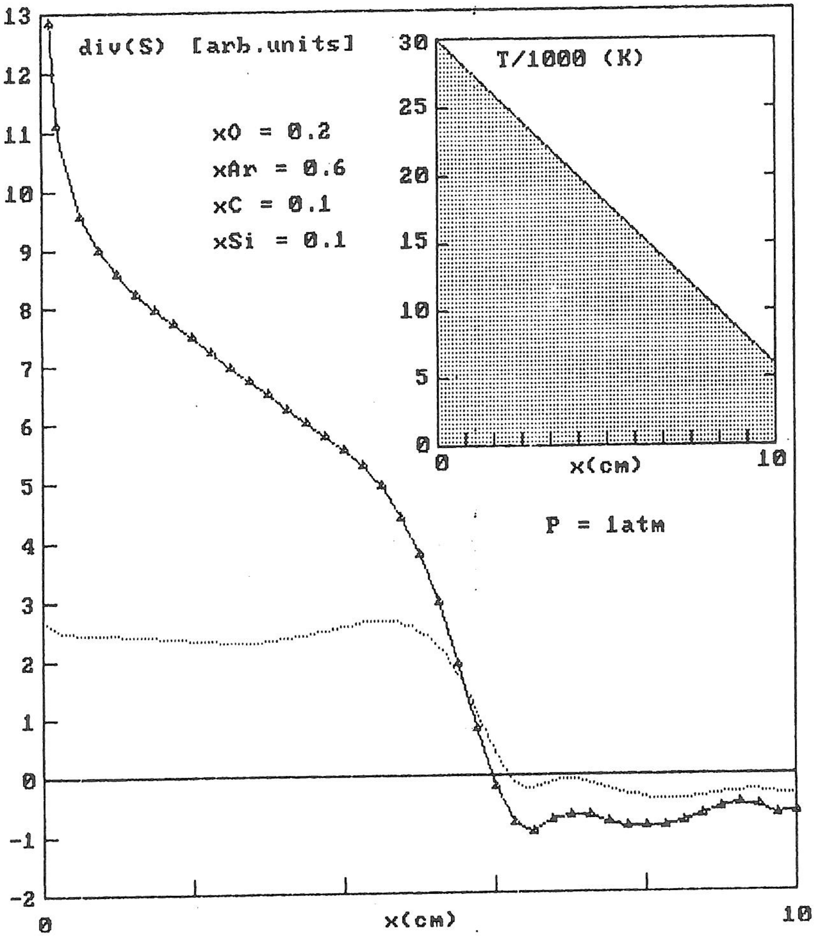

Fig.4

In Fig.4 by dotted line presented also the divergence of radiative flux in the continuous spectrum. All the spectral line are removed, but all the processes which are forming the continuous spectrum (photoionization, free-free transitions, molecular bands,...) are taken into account. This illustration presents the role of spectral lines in the radiation transfer in plasma.

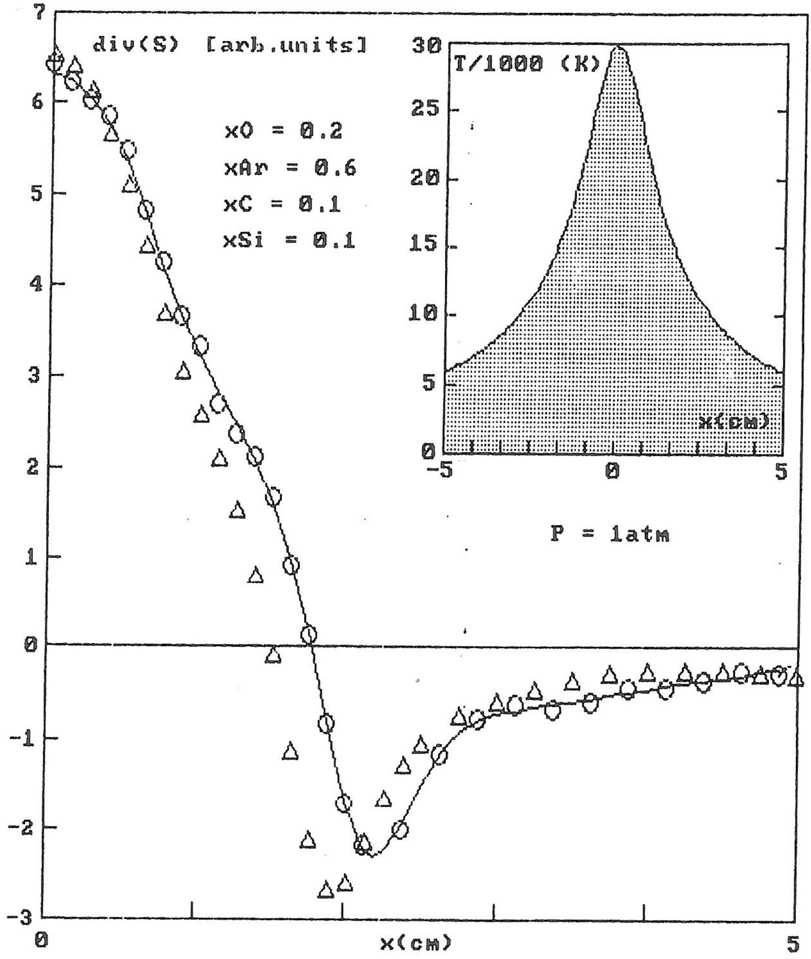

Fig.5

In Fig.5 and Fig.6 the results of calculations of divergence of radiative flux are represented for the two temperature profiles (more filled and less filled) which are the typical for free-burned electric arcs in plasma reactors. The results of exact calculations are represented by solid lines and the results of the PCh approximations are represented by triangles for three-parameters approximation (Ts, Tx, x) and by circles for many-temperature approximation. Using of two referense temperatures gives satisfactory estimation. Using of many-temperature approximation gives practically exact results.

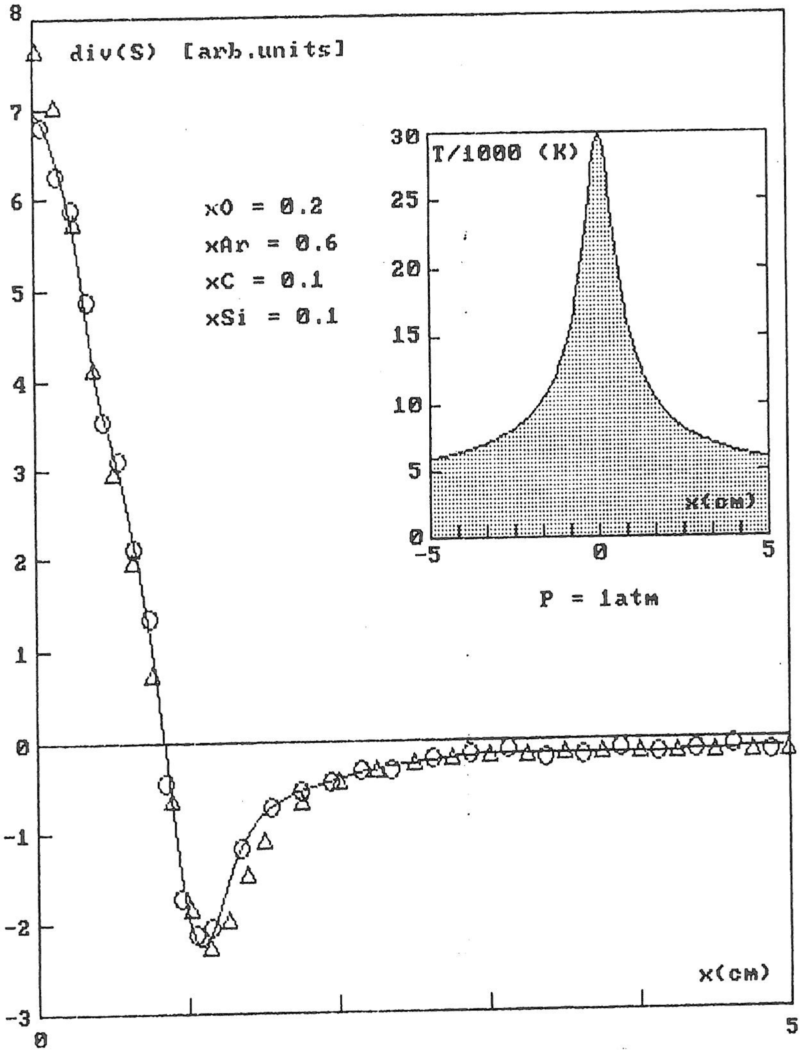

Fig.6

Comparatively small efforts which are needed for increase of the number of reference temperatures allow to decide practically a problem of exact and fast calculations of radiative fluxes and their divergences. The accuracy of calculation of radiative transfer will be determine only by the accuracy of initial data for elementary processes (transition probabilities, photoionization cross-sections, widthes of spectral lines and so on).

V. SUMMARY

Using of the method of Partial Characteristics with many-temperature approximation allows to get practically exact values of radiative fluxes and their divergences.

The preparation of many-temperature arrays of Partial Characteristics is not more difficult than the preparation of two-temperature ones.

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