AN EMPIRICAL CORRELATION FOR THE DRAG FORCE ON A CYLINDER EXPOSED TO A PLASMA CROSS FLOW

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ABSTRACT: An experimental study has been conducted for the drag force acting on a cylinder exposed to a plasma cross-flow within the continuum regime. The drag forces are measured by sweeping a cylindrical wire across an axisymmetrical argon plasma jet as a function of the lateral distance of the wire with respect to the plasma-jet axis. By using the Abel inversion, the local drag force at the jet axis and thus the drag coefficient are obtained. It is shown that the measured drag forces can be well correlated by using the standard drag relation with the film temperature as the reference temperature to evaluate the gas density and viscosity in the Reynolds number.

I. INTRODUCTION

Many expressions for the drag force acting on a single particle within continuum regime have been proposed in the literature due to the importance of the drag expression in the modeling of thermal plasma processing of particulate matter\textsuperscript{[1–3]}. None of them is generally accepted by the investigators working in this field. More experimental and theoretical studies are thus required. The present paper is concerned with an experimental study of the drag force acting on a cylinder exposed to an argon plasma cross-flow.

The drag force acting on a cylinder exposed to a steady isothermal cross-flow has been investigated extensively, and the standard experimental drag curve has been given in many textbooks of fluid mechanics, showing the drag coefficient as a function of the Reynolds number (e.g. Schlichting\textsuperscript{[4]}). Empirical expressions have been suggested to approximate the standard drag curve for a cylinder \(f(Re)\) by Clift et al. \textsuperscript{[5]} as follows:

\[
\begin{align*}
C_D &= C_D'(1 + 0.147 Re^{-0.82}) & (0.1 \leq Re \leq 5) \\
C_D &= C_D'(1 + 0.227 Re^{-0.55}) & (5 \leq Re \leq 40) \\
C_D &= C_D'(1 + 0.0838 Re^{-0.82}) & (40 \leq Re \leq 400)
\end{align*}
\]

where

\[
C_D' = 9.689 Re^{-0.78}
\]  

Under thermal plasma conditions, although a few experimental studies have been
reported about the drag force acting on a sphere\textsuperscript{[3,6,7]}, so far no experimental data or theoretical expressions are available in the literature concerning the drag force acting on a cylinder. The present study intends to report some experimental data of the cylinder drag force obtained in our Lab. An empirical correlation is also proposed.

II EXPERIMENTAL SETUP AND METHOD

The experimental setup and experimental method are similar to those used in the previous studies concerning the heat transfer\textsuperscript{[8]} and the drag force\textsuperscript{[3]} on a sphere exposed to a thermal plasma flow. Namely, an argon plasma jet is produced by using a high intensity d.c. arc burning between a central tungsten cathode and a copper anode-nozzle in a plasma generator. The arc current is in the range of 80 -- 250 A and the arc voltages 16 -- 20 V. The cathode diameter is 6 mm, while the inside diameter of the exit of the anode-nozzle is 8 mm. Argon is introduced tangentially into the plasma-jet generator and issues upward from the anode-nozzle into the room-temperature air surroundings after being heated by the arc. The flowrate of argon varies within the range of 0.34 -- 0.68 STP m\textsuperscript{3} / hr in order to keep the plasma jet within the laminar regime associated with extremely low noise level. All the cylinder drag measurements are conducted at the measuring section which is located 3.5 mm above the exit section of the anode-nozzle. At the measuring section, plasma temperature and velocity distributions are measured respectively by using the absolute spectral-line intensity method and by a movable water-cooled Pitot tube as before\textsuperscript{[8]}.

The drag measuring system employed in this study is shown in Fig.1, which is also similar to that used previously in the study of sphere drag\textsuperscript{[3]} except that a cylindrical wire is used to substitute for the sphere probe. The drag measuring system consists of a cylinder wire 1, on which the drag force is to be measured, a supporting wire 2 welded with the cylinder probe, an elastic steel piece 3 and a sensitive displacement sensor 4. The length of the cylinder wire is considerably greater than the radius of the plasma jet in the measuring section which is 3.5 mm above the generator exit. The upper end of the supporting wire is cemented to a thin steel piece 3, which is far away from the plasma jet and is thus less influenced by the jet flow. As the drag probe is exposed to the argon plasma jet, the drag force on the cylinder wire causes an elastic deformation of the thin steel piece, which is measured by the sensitive displacement sensor 4 (0.8 mV output per 1 \( \mu \)m deformation). In order to obtain the relationship between the magnitude of drag force and the output electric voltage of the displacement sensor, a calibration of the displacement sensor.

Figure 1. Schematic diagram of the drag measuring system
sensor is needed. For the calibration, an additional wire was put on the cylinder wire at the position which would pass through the jet axis when the plasma-jet generator is moved to sweep across the drag probe. By cutting successively this additional wire and thus varying the probe-wire weight, a calibration curve was obtained, showing a fairly good straight line. By using this calibration relation, the drag force on the drag probe is readily calculated from the output signal of the displacement sensor.

Due to the extremely high sensitivity of the displacement sensor to any external shock or vibration, we move the plasma-jet generator to sweep across the drag probe when the drag experiments were conducted, while the drag probe, the elastic steel piece, and the displacement sensor remain stationary. The sweeping speed must be controlled to be high enough to avoid the melting of the drag probe, but be low enough to meet the requirement raised by the response time of the recording instrument and by the relaxation time of the boundary layer around the probe. In the present study, the sweeping speed is about 0.1 m/s. The relaxation time of the boundary layer around the cylinder probe is estimated by a special computational study to be \(10^{-4}\) s, and thus is negligible in comparison with the dwell time of the drag probe in plasma flow. In the sweeping process, the axis of the cylinder wire is always kept at the measuring section and is thus normal to the geometric axis of the plasma jet.

By sweeping the drag probe across the axisymmetrical plasma jet, the drag force on the whole cylinder probe, \(F(x)\), can be measured as the function of the lateral distance, \(x\), from the jet axis. The accumulative drag force \(F(x)\) can be expressed as

\[
F(x) = 2 \int_{r}^{R} \frac{f(r)rdr}{\left(r^2 - x^2\right)^{1/2}}
\]  

Where \(r\) is the radial distance from the jet axis and \(R\) is the jet radius at the measuring section. Equation (5) is recognized as the Abel integral equation whose solution is

\[
f(r) = \frac{1}{\pi} \int_{r}^{R} \frac{F'(x)dx}{\left(x^2 - r^2\right)^{1/2}}
\]

Where \(F'(x)\) is the derivative of \(F(x)\) with respect to \(x\). Through the numerical integration of Equation (6), the radial distributions of the local drag force on the cylinder wire for unit cylinder length, \(f(r)\), is easily calculated as a function of the radial position \(r\). However, only the drag force value at the jet axis, \(f(0)\), is adopted by us in order to exclude the effect due to the entrainment of the ambient air into the argon plasma or due to the possible existence of a non-LTE plasma state in the region near the edge of the plasma jet. The drag coefficient can be calculated by

\[
C_D = \frac{f(0)}{(1/2) \rho \pi V^2 \omega d}
\]

Where \(d\) is the cylinder diameter, and \(\rho\), \(\rho_\infty\), and \(V\), \(V_\infty\) are the oncoming plasma density and velocity, respectively.

The experimental procedure is as follows: For a fixed combination of the cylinder diameter and the plasma parameters, after the plasma jet generator has been operated steadily, the plasma temperature and velocity distributions at the measuring section are measured. Drag forces on the cylinder probe are then measured by sweeping the
plasma-jet generator across the stationary drag probe while the cylindrical wire is kept at the measuring section. Sometimes the arc voltage may abruptly assume a change up to 1 V due to a random variation of the arc-root location at the anode surface, resulting in an appreciable variation of the plasma temperature. Drag-force data are adopted only as the plasma parameters remain unchanged before and after the drag force is measured. By measuring the drag force on the whole cylindrical probe $F(x)$ and by using the Abel inversion, the value of the drag force for unit cylinder length at the jet axis, $f(0)$, and the drag coefficient $C_D$ can be obtained by using Eqs.(5)–(7).

III. RESULTS AND DISCUSSIONS

Using the sweeping method, we measured the cylinder drag forces for different cylinder diameters (0.3-1.3 mm), arc currents (80-200 A) and argon flow rates (0.34-0.68 STP m$^3$/hr). For a typical case with the arc current 150 A and argon flow rate 0.52 STP m$^3$/hr, the maximum temperature and velocity are 10800 K and 82 m/s, respectively, at the plasma-jet axis. Typical records of the accumulative drag force $F(x)$ experienced by the cylinder wire assume fairly good axisymmetry, and thus satisfy the prerequisite for the Abel inversion to be employed.

Under the experimental conditions with a temperature difference as great as $10^4$ K, the Reynolds number based on the plasma density and viscosity evaluated at oncoming plasma temperature, $Re_\infty$, is much less than that evaluated at the cylinder surface temperature, $Re_W$. Hence, it is expected that the experimental drag forces would be less than that predicted by the standard drag curve for the isothermal flow in the figure $C_D$ versus $Re_\infty$, while an inverse tendency would appear in the figure $C_D$ versus $Re_W$. Fig.2 and Fig.3 do indeed demonstrate this expectation. Experimental data of the cylinder drag coefficient are shown in Figure 2 as the function of the Reynolds number $Re_\infty$ evaluated based on the oncoming plasma density and viscosity ($Re_\infty = \rho_\infty v_\infty d/\mu_\infty$) for 36 different combinations of the cylinder diameter and the plasma parameters, where $\mu$ is the gas viscosity. In Fig.2, each experimental point represents an averaged value of several drag measurements repeated at the same experimental conditions. The standard drag curve $f(Re_\infty)$ expressed by Eq.(1)–(4) is also shown in this figure. As seen, the experimental results show that the measured drag coefficients or drag forces are always less than their counterparts at the standard drag curve for the same Reynolds numbers $Re_\infty$. For example, experimental value of $C_D$ at $Re_\infty=3$ is about 50% of its counterpart read from the standard drag curve $f(Re_\infty)$, while the experimental value of $C_D$ at $Re_\infty=17$ is about 70% of the standard curve value. It is also seen that as the Reynolds number $Re_\infty$ is within the range of 3--17, with the increase of $Re_\infty$, the difference decreases between the experimental drag value under plasma conditions and that at the standard drag curve $f(Re_\infty)$. In Fig.3, the measured cylinder drag coefficients are replotted as the function of the Reynolds numbers $Re_W$ with gas density and viscosity evaluated at the surface temperature of the cylinder (500 K) and are compared with corresponding standard drag curve $f(Re_W)$. As seen, all the drag data locate above their counterparts at the standard drag curve
Figure 2. Drag coefficient versus the Reynolds number $Re_{\infty}$.

$\mathcal{f}(Re_W)$ for the same Reynolds numbers $Re_W$.

It has been found that the film temperature $T_f = (T_{\infty} + T_W) / 2$ is the most appropriate reference temperature to correlate the present experimental data, where $T_W$ and $T_{\infty}$ are the cylinder surface temperature and oncoming plasma temperature, respectively. In Fig.4, the 36 experimental drag data are replotted as the function of the Reynolds number $Re_f$ with gas density and viscosity evaluated at the film temperature. In this figure, the standard drag expressions for the isothermal flow using $Re_f$, i.e. $\mathcal{f}(Re_f)$, have been shown to be able to correlate the experimental data very well. Hence, under the present experimental conditions, the following empirical correlations can be employed to represent the drag force acting on a cylinder in a thermal plasma cross-flow:

$$C_D = C_{DF}'(1 + 0.147 Re_f^{0.82}) \quad (0.1 < Re_f \leq 5) \quad (8)$$

$$C_D = C_{DF}'(1 + 0.227 Re_f^{0.55}) \quad (5 < Re_f \leq 40) \quad (9)$$

$$C_D = C_{DF}'(1 + 0.0838 Re_f^{0.82}) \quad (40 < Re_f \leq 400) \quad (10)$$

in which

$$C_{DF}' = 9.689 Re_f^{-0.78} \quad (11)$$

1011
while the Reynolds number is calculated by using the gas density and viscosity evaluate at film temperature:

\[ \text{Re}_f = \frac{\rho \alpha V_x d}{\mu_x} \]  

(12)

This result is somewhat unexpected, because the drag correlation \( f(\text{Re}_f) \) is often considered by many researches to be unsatisfactory under thermal plasma conditions. It should be noted the plasma temperature range (9990−11370 K) in the present experiment is comparatively narrow. It is expected that the plasma jet is always not completely axisymmetrical. Using the Abel inversion for this case in the processing of experimental data may introduce some error. Further studies are highly desirable.

Some factors which may affect the wire drag force were also investigated in the experiment. It was found that increasing the sweeping speed from 0.1 m/s to 0.2 m/s does not affect the drag value. By using different wire diameters (0.30, 0.50, 0.87 and 1.30 mm), it was found that the wire drag forces were proportional to the square root of the wire diameter. When different negatively biased voltages were applied between the wire and anode-nozzle, it was found that the biased voltage has no effect on the drag force.

IV. CONCLUSIONS

An experimental study has been conducted to measure the drag force acting on a cylinder immersed into an argon plasma cross-flow. Experimental data show that the cylinder drag correlation well known for the isothermal flow can also be employed for calculating the cylinder drag under thermal plasma conditions, provided that the film temperature \( T_f = (T_a + T_w) / 2 \) is employed to evaluate the gas density and viscosity in the Reynolds number (i.e., \( \text{Re}_f = \rho \alpha V_x d / \mu_f \)) appearing in \( f(\text{Re}_f) \).

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