SELF-CONSISTENT MODELLING OF ARC-CATHODE INTERACTION
IN HIGH-PRESSURE ARC-DISCHARGES

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Abstract

A numerical model describing the interaction of the arc plasma with a hot thermionic cathode is presented. The model includes two modules, one calculating the near-cathode plasma layer and another calculating the temperature distribution in the cathode body. The model of the near-cathode plasma layer is based on the conventional concept that the current transfer through the layer is one-dimensional. The temperature distribution in the cathode body is governed by the heat conduction equation, the boundary condition at the cathode surface being the condition of energy balance (the density of the energy flux from the plasma equals sum of the density of the heat flux removed by thermal conduction into the cathode body and of radiation losses). The resulting 2D nonlinear boundary-value problem is solved by iterations with the use of a finite-difference method with direct solving of finite-difference equations. Results of calculations for conditions of a model arc lamp are presented and found to agree with experimental data.

2. Introduction

The problem of interaction of high-pressure arc plasmas with cathodes continues to receive a great deal of attention in the literature, both due to being of considerable scientific interest and due to the necessity to develop calculation methods which could be used by industry. For example, understanding of high-pressure arc plasma-cathode interaction is of crucial importance for identifying factors that affect the performance and lifetime of high-intensity discharge lamps [1].

A simple model of current transfer to hot arc cathodes may be obtained on the basis of the conventional concept that the energy flux coming from the plasma to the cathode surface is generated in a thin near-cathode plasma layer. Then the problem may be split. First, one calculates the current transfer through the near-cathode layer, which is one-dimensional and is governed by values of the local surface cathode temperature and of the near-cathode voltage drop, which takes the same value at all points of current-collecting surface of the cathode. After this problem has been solved, one can solve thermal conduction equation for the temperature distribution inside the cathode body, using as a boundary condition the dependence of the energy flux from the plasma to the cathode surface on the local surface temperature, found at the first step. After this equation has been solved, one can determine, in particular, the current density distribution over the cathode surface.

This model has been known for many years; e.g., [2] and references therein. In recent
years, interest in this problem reappeared (e.g., [3-6]) after it has been shown [3,4] that the problem has multiple solutions, some of them describing modes with the external heat flux being distributed over a large fraction of the cathode surface (diffuse modes) and others describing modes with the heat flux localized in small areas (spot modes).

In [3,4], cathodes in the form of a parallelepiped and of a right circular cylinder were treated neglecting the current collection by the lateral surface of the cathode (i.e., the lateral surface of the cathode was considered thermally and electrically insulated). The bifurcation analysis was employed and asymptotic solutions in the vicinity of the bifurcation points was found for different kind of spot modes of current transfer to hot arc cathodes. Cathodes of more realistic configurations were treated numerically in [5,6]. In the present work, further numerical work is performed. Results of numerical calculations for conditions of a model arc lamp [7-9] are given and found to compare favorably with experimental data.

3. Model and numerics

Let us consider a cathode made of a substance with the thermal conductivity \( \kappa \) being a known function of the temperature, \( \kappa = \kappa(T) \). A part \( \Gamma_h \) of the cathode surface is heated by an energy flux coming from the plasma and a part \( \Gamma_e \) is maintained at a fixed temperature \( T_e \) by external cooling. Neglecting the Joule heat production inside the cathode body, a steady-state temperature distribution in the cathode is determined by the nonlinear boundary-value problem for the Laplace equation,

\[
\nabla^2 \psi = 0.
\]

\[
\Gamma_h : \frac{\partial \psi}{\partial n} = q(\psi, U), \quad \Gamma_e : \psi = 0.
\]

Here \( n \) is a direction locally orthogonal to the current-collecting surface and directed outside the cathode, the function \( \psi = \psi(T) \) is the energy flux potential related to the temperature by the equation

\[
\psi(T) = \int_{T_e}^{T} \kappa(T) dT.
\]

\( q(\psi, U) \) is the density of the net energy flux from the plasma to the cathode surface, and \( U \) is the near-cathode voltage drop (a constant parameter which should be chosen is such a way that the integral current to the cathode surface take a prescribed value).

The dependence \( q(\psi, U) \) (the local density of the net energy flux from the plasma to the cathode surface as a function of the local surface cathode temperature and of the near-cathode voltage drop) is calculated by means of analysis of a plasma layer adjacent to the cathode surface and is considered as known while treating the problem of heat conduction in the cathode body.

The same applies to the dependence \( j(\psi, U) \), describing the local electric current density from the plasma to the cathode surface as a function of the local surface cathode temperature.
and of the near-cathode voltage drop. After the problem (1), (2) has been solved, one can substitute the calculated surface temperature distribution into the dependence \( j(\psi, U) \), thus finding the electric current density distribution over the current-collecting surface of the cathode. Integrating the latter, one finds the total arc current corresponding to the value of the near-cathode voltage drop being considered. Solving the problem for varying \( U \), one can obtain solutions describing the whole current-voltage characteristic of the cathode.

The nonlinear boundary-value problem (1), (2) was solved for the case of axially symmetric cathode. An iterative approach was used. With respect to solving a (linearized) problem on each iteration, we have found that iterative methods such as successive overrelaxation (SOR) are not robust enough. On the other hand, direct methods commonly used such as fast Fourier transform (FFT) or a combination of Fourier analysis and cyclic reduction (FACR) are inapplicable to the considered problem since variables do not separate. Therefore, a special method of direct solving of finite-difference equations was chosen.

The functions \( q(\psi, U) \) and \( j(\psi, U) \) were calculated by means of the approach [10], supplemented with account of the radiative cooling of the cathode surface. In this approach, it is assumed that the dominating contribution to the energy flux from the plasma to the cathode surface is given by ion bombardment, while the dominating cooling mechanism is cooling due to thermionic emission. Input parameters for calculations were plasma-producing gas and its pressure, cathode geometry and material, and near-cathode voltage; all other parameters are found, including temperature distribution in the cathode body and on the surface, the current density distribution over the cathode surface, the arc current, the power removed by heat conduction inside the cathode body, the power irradiated from the cathode surface.

Special care was used to obtain initial approximations for solutions corresponding to different modes of current transfer to the cathode. After a solution for a certain variant has been found, solutions for other variants in the same mode can be obtained by moving in the parameter space.

5. Results and discussion

Calculations reported below have been performed for conditions of a model high-intensity discharge lamp [7-9]. In these experiments, the cathode was in the form of a circular cylinder made of tungsten or thoriated tungsten; the plasma-producing gas was argon and its pressure was 2.6 atm.

The modelling results are shown in Figs. 1-8; here \( R \) is the radius of the cathode, \( h \) is its height, \( r \) is the distance from the center of the cathode, and \( z \) is counted from the top of the cathode (which is in contact with the plasma) to the bottom (which is cooled). Also shown in Figs. 1-3 are experimental data [7-9]. One can see that the agreement between the modelling and the experiment is good in all the cases.

One can see from Fig. 1 that the radiation cooling of the cathode is more significant than the power removal due to heat conduction inside the cathode body, which is a consequence of large values of the ratio \( h / R \) for the cathodes considered.

The cathode surface temperature weakly increases with increasing current, see Fig. 2. The temperature is nearly independent of the cathode length. As it could be expected, the surface temperature increases with an increase of the work function of the cathode material. In summary, one can say that the cathode surface temperature depends only weakly on the cathode design and on the arc current, while an effect of the work function of the cathode material is rather strong. This appearance is due to the fact that the function \( q(\psi) \) for a given
$U$ is a strong function of the temperature [4,10], which is a consequence of the Arrhenius character of the processes involved.

One can see from the current-voltage characteristics shown in Fig. 3 that the voltage drop decreases with an increasing current, however the power input into the cathodic part of the arc, equal to $IU$, increases. The latter is consistent with the above-mentioned increase of the surface temperature. The voltage drop (and, accordingly, the power consumption) increases with an increase of the work function of the cathode material.

The line 1 in Fig. 4 represents a current-voltage characteristic of spot mode with a spot at the center of the cathode. Line 2 represents a current-voltage characteristic of diffuse mode. Both characteristics are quite close, however the temperature and current density distributions over the front surface of the cathode are essentially different (see Figs. 5 and 6). In both modes, an increase of current is accompanied by an increase of the power input in the cathode, which results in the increase of the temperature. The temperatures in the spot are above the melting point of tungsten, and the current densities are in excess of $10^8$ A/m².

Also shown in Fig. 4 are current-voltage characteristics of the diffuse mode calculated without account of both the current collection by (and the energy flux from the plasma to) the lateral surface of the cathode and the radiative cooling of the cathode surface (line 3) or without account of the radiative cooling only (line 4). One can see that the current-voltage characteristics of these two variants are very close.

In Fig. 7, the distribution of the current density along the lateral surface of the cathode is shown, calculated without account of the radiative cooling (but, obviously, with account of the current collection by the lateral surface). As it should have been expected, the current density is significant only in the region close to the top of the cathode, where the temperature is still high. One can conclude that although the current-voltage characteristics of the variants with or without account of current collection by the lateral surface of the cathode are very close, the current distributions along the cathode surface are essentially different: in the case with current collection by the lateral surface, the current collected by the front and lateral surfaces of the cathode is, respectively, 0.6 A and 1.4 A for $I = 2$ A; 1.7 A and 4.3 A for $I = 6$ A. Note that the same applies to the power balance: although the total power removed by heat conduction inside the cathode body for these two variants is very close (~46.9 W and 47.8 W with and, respectively, without the current to the lateral surface for $I = 2$ A;...
Fig. 3. Current-voltage characteristics. Diffuse mode. Lines: modeling. Points: experiment [7].

Fig. 4. Current-voltage characteristics of different modes of current transfer to a tungsten cathode.

Fig. 5. Distribution of temperature along the front surface of a tungsten cathode. $h = 20$ mm, $R = 1$ mm.

Fig. 6. Distribution of current density along the front surface of a tungsten cathode. $h = 20$ mm, $R = 1$ mm.

Fig. 7. Distribution of current density along the lateral surface of a tungsten cathode. $h = 20$ mm, $R = 1$ mm.

Fig. 8. Distribution of temperature along the lateral surface of a tungsten cathode. $h = 20$ mm, $R = 1$ mm.
50.3 W and 51.3 W for I = 6 A), this power is supplied in essentially different ways: in the case with the current to the lateral surface, the power removed by heat conduction from the front and lateral surfaces of the cathode is, respectively, 13.7 W and 33.2 W for I = 2 A; 14.1 W and 36.2 W for I = 6 A.

Distribution of the temperature along the lateral surface of the cathode, calculated without account of the radiative cooling and with or without account of the current collection by the lateral surface of the cathode, is shown in Fig. 8. In the case without account of the current collection by the lateral surface, the temperature variation along the cathode height is linear except near the bottom of the cathode (the latter is due to the temperature dependence of the thermal conductivity of the cathode material). In the case with account of the current collection by the lateral surface, the temperature variation along the cathode height is nonlinear also near the top of the cathode, which is due to the power supply through the lateral surface.

The above suggests that the power balance of the cathode is not strongly affected by a distribution of this power over the top of the cathode. Of course, this is a consequence of small values of the ratio $R/h$ for the cathode considered.

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References