Low frequency filamentation of a plasma

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Abstract

The effect of thermal motion of charged particles in the filamentation of a nonrelativistic current-driven plasma in the diffusion and ion-acoustic frequency regions is investigated. The period and the establishment time of the filamentation structure and the threshold for instability development are obtained.

1. Introduction

Filamentation in laser-plasma and beam-plasma systems as well as being a fundamental issue has been the subject of a great activity due to the numerous applications including x-ray lasers and laser-plasma-produced accelerators. The ordinary-mode filamentation instability in the aforementioned systems has been investigated in circumstances where a low-density electron beam is injected into a high-density preionized plasma under the special instability criterion. But, when a low-density electron beam is injected into a dense neutral gas under the special instability criterion, the beam is experimentally observed not to filament. The absence of filamentation instability in circumstances where the beam-plasma system is not neutralized, strongly, suggests that azimuthal self-magnetic field has a stabilizing influence on the filamentation instability. Moreover, in earlier works, it was shown that the interchange instability corresponding to the laminarization of the electron beam into separate current-carrying filaments is possible in a plasma-beam system in the absence of an external magnetic field and thermal motion of particles.

In this paper, we consider the low frequency kinetic effects, i.e., particles thermal motion and the plasma kinetic pressure which resists the layer current compression. Therefore, the instability, developed in the beam-plasma system, should have a threshold. The consideration of thermal motion allows us to determine the threshold for the appearance of the pinch instability and the period of cross structure, resulted from the instability development.

2. Diffusive kinetic effects in filamentation

We now consider a nonrelativistic current-carrying plasma, i.e., a plasma with a small group of electrons with nonrelativistic velocity moving through a medium of resting particles,
or, equivalently, a plasma with a nonrelativistic electron beam injected into it. We assume that the beam density to be smaller than the plasma density. Furthermore, we suppose that both the electrons of the beam, in their intrinsic frame, and the particles of the bulk of the plasma have Maxwellian distributions with nonrelativistic temperature. To calculate the dielectric permittivity of this system in the laboratory frame, without solving the kinetic equation, one can apply the Lorentz transformation.

We now consider the particle thermal motion. Consequently, the plasma pressure which resists the layer current compression should be considered also. The consideration of thermal motion allows us to determine a threshold for the appearance of the pinch instability and to obtain the period of cross structure arisen from the instability development as well.

We consider the diffusion frequency region which is relevant for discharge plasma: 

$$\omega, k \ll \omega_0, \nu_0. \quad (1)$$

Using the dielectric permittivity tensor

$$\varepsilon_0^0(\omega_0, k) = \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \varepsilon_0^0(\omega_0, k) + \frac{k_i k_j}{k^2} \varepsilon_1^0(\omega_0, k)$$

(2)

where $\varepsilon_0^0(\omega_0, k)$ and $\varepsilon_1^0(\omega_0, k)$ are, respectively, the longitudinal and transverse dielectric permittivity of the $\alpha$ species in an isotropic plasma, in the diffusion frequency region, for $u^*_\alpha = \tilde{u}$ and $\omega_{\infty} = 0$, we obtain

$$\varepsilon_1^\alpha(\omega_0^\alpha, k) = 1 + i \frac{\omega_0^{\alpha 2}}{(\omega - \tilde{k} \cdot \tilde{u})} \nu_\alpha, \quad \varepsilon_0^\alpha(\omega_0^\alpha, k) = 1 + i \frac{\omega_0^{\alpha 2}}{\omega \nu_\alpha}. \quad (3)$$

The dispersion equation for transverse oscillation, i.e., $\tilde{k} \cdot \tilde{u} = 0$, can be written as:

$$\left( k^2 - \frac{\omega_0^{\alpha 2}}{\nu_\alpha} \right) \left( 1 + i \frac{\omega_0^{\alpha 2}}{\omega \nu_\alpha + i k^2 \omega \nu_\alpha} + i \frac{\omega_0^{\alpha 2}}{\omega \nu_\alpha + i k^2 \omega \nu_\alpha} \right) +$$

$$\left( k^2 \nu_\alpha^2 \frac{\omega_0^{\alpha 2}}{\nu_\alpha} \right) \frac{\omega_0^{\alpha 2}}{\omega \nu_\alpha + i k^2 \omega \nu_\alpha} \nu_\alpha + i k^2 \nu_\alpha^2 = 0 \quad (4)$$

where in the static limit, $\omega \to 0$, we obtain

$$k^2 = k_0^2 = \left( \frac{u^2}{c^2} \frac{\omega_0^{\alpha 2}}{\nu_\alpha + i k^2 \nu_\alpha} \right). \quad (5)$$

In the above equation $\nu_\alpha = \sqrt{T_\alpha / M}$ is the ion sound velocity; $T_\alpha(T_\star)$ is $\alpha$ species (electron) temperature and $M$ is the ion mass. Equation (4) indicates that in a strong collision plasma a transverse structure with a characteristic period $l_0 = \pi / k_0$ can exist in the static limit.

As we have a current in the plasma, a magnetic field $B_0 \approx 4 \pi n u x / c$ arises around its axis where $x$ is the lateral distance from current axis and $n$ is the current density. This field can compress the plasma layer if the transverse plasma dimension is greater than $x_0$. Therefore, in this case magnetic pressure is larger than gas kinetic pressure:

$$\frac{B_0^2}{8 \pi} = \frac{1}{8 \pi} \left( \frac{4 \pi n u x_0}{c} \right)^2 \gg n_0 (T_e + T_i). \quad (6)$$

826
where \( n_0 \) is the plasma density. From Eq. (13) we can find

\[
x_0 = \frac{2\varepsilon}{u} \sqrt{\frac{\nu_o^2 + n_0^2}{\omega_{tr}^2}} \approx \lambda_0,
\]

which singles out the pinch nature of the plasma transverse structure. The time, needed for the establishment of this structure, can be determined from the time which is necessary for instability development. To obtain this value, we consider a system close to the threshold \( k \approx k_0 \). Then, from the Eq. (4) we find

\[
\omega = -i \left[ k^2 \left( \frac{\nu_i^2 + \nu_o^2}{\nu_i} \right) - \frac{u^2 \omega_{tr}^2}{c^2 \nu_i} \right] = -i \frac{k^2}{\nu_i} \left( \frac{\nu_i^2 + \nu_o^2}{\nu_i} \right) \left( 1 - \frac{k^2 \nu_i}{k^2} \right).
\]

The first term in Eq. (8) describes the diffusing attenuation of the inhomogeneity of a layer with thickness \( \approx 1/k \), while the second term describes the compression resulting from the magnetic field. The compression takes place when \( k < k_0 \) or when the lateral dimension of the plasma is greater than \( 1/k_0 \). This means that the current layer will be subdivided into separate current filaments where its period of structure is of the order of \( x_0 \). Finally, the inhomogeneity determines the time of the structure establishment where its minimal value can be determined from Eq. (8):

\[
\tau \approx \frac{1}{3 \lambda_0} \approx \frac{c^2 \nu_i}{u^2 \omega_{tr}^2}.
\]

Here it should be noted that the studied phenomenon may purely appear under the conditions of slight warming up of electrons in the plasma where the effects of instability overturns the plasma (e.g. overheat ionization) and in nonelectronegative gases where the adhesive-tack instabilities do not take place. In this case, \( u \approx 10^2 \text{ cm/s} \) and \( n_0 \approx 10^{14} - 10^{15} \text{ cm}^{-3} \) for \( P_0 \geq 1 - 10 \text{ torr} \) and \( T \leq 10^4 K \), for light gases like hydrogen. Hence, if \( \nu_i \approx 10^5 - 10^7 \text{s}^{-1} \) and \( \omega_{tr} \approx 3 \times 10^{10} \text{s}^{-1} \), we find \( \tau \approx 10^{-4} - 10^{-3} \text{s} \) and \( \lambda_0 \approx 3 - 5 \text{ cm} \). In the earth's magnetosphere with ion densities \( n_i \leq 10^4 \text{ cm}^{-3} \), for \( P_0 \approx 10^4 \text{ cm}^{-3} \), the solar wind velocity \( u \approx 10^6 \text{ cm/s} \), \( T \approx 10^6 K \), \( \nu_{tr} \approx 10^6 \text{ cm/s} \), \( \nu_i \approx 10^{-6} \text{ s}^{-1} \) and \( \omega_{tr} \approx 10^3 \text{ s}^{-1} \), we obtain \( \tau \approx 1 - 10 \text{s} \) and \( \lambda_0 \approx 10 \text{ km} \).

3. Ion-acoustic kinetic effects in filamentation

Now we consider the ion-acoustic frequency region

\[
\nu_i, k\nu_i, \ll \omega \ll k\nu_{tr} \ll \nu_e.
\]

Making use Eq. (2), in the aforementioned frequency region, for \( \bar{u}_e = \bar{u} \) and \( \bar{u}_{ion} = 0 \), we obtain

\[
\varepsilon''_n(\omega, k) = \varepsilon''_t(\omega, k) = 1 - \frac{\omega^2}{\omega_{tr}^2};
\]

\[
\varepsilon''_n(\omega, k) = 1 + \frac{\omega^2}{\omega_{tr}^2} \quad \varepsilon''_t(\omega, k) = 1 + \frac{\omega^2}{\omega_{tr}^2 + \nu_e^2} + \frac{\omega^2}{\nu_e^2}.
\]

The dispersion equation of transverse oscillation, i.e., \( \ddot{\bar{u}} = 0 \) can be written as

\[
\varepsilon_{11} \left[ \frac{k^2}{c^2} - \frac{\omega^2}{c^2} \varepsilon_{22} \right] + \frac{\omega^2}{c^2} \varepsilon_{13} = 0.
\]
where

\[ \varepsilon_{11} = \varepsilon_1^m + \varepsilon_1^h - 1 , \quad \varepsilon_{12} = \frac{k u}{\omega} \left[ \varepsilon_1^m - 1 \right] , \]

\[ \varepsilon_{22} = \varepsilon_1^m + \varepsilon_1^h - 1 + \frac{k^2 u^2}{\omega^2} \left[ \varepsilon_1^m - 1 \right] . \] (13)

We, firstly, analyze the dispersion equation (12) in the short wavelength limit under the condition \( \omega \gamma_c \ll k^2 \gamma_c^2 \). In this case, in the static limit, \( \omega \rightarrow 0 \), we obtain the frequency spectrum

\[ k^2 \equiv k_1^2 = -\frac{\omega^2}{2 \nu_1^2} , \] (14)

for a dense plasma. This spectrum indicates that in a strong collisional dense plasma a transverse structure can not occur in the static limit in the ion-acoustic frequency region, i.e., filamentation instability does not happen. However, from dispersion Eq. (19) for a not too dense plasma we find the frequency spectrum

\[ \omega^2 = \omega_{\gamma_c}^2 \left( 1 - \frac{k^2}{k_2^2} \right) , \quad k_2^2 = \omega_{\gamma_c}^2 / 2 \nu_2^2 . \] (15)

This spectrum indicates that in the static limit a transverse structure can occur and filamentation instability may arise when the wave length of the ion-acoustic wave is smaller than the quantity \( 2 \nu_2^2 / \omega_{\gamma_c}^2 \).

We now consider the long wavelength limit, \( \omega \gamma_c \gg k^2 \gamma_c^2 \). In this case, from Eq. (12) we obtain the frequency spectrum

\[ \omega^2 = \omega_{\gamma_c}^2 \left( 1 - \frac{\nu_1^2}{k^2 \gamma_c^2 + \nu_1^2} \right) . \] (16)

Here, it is clear that in the static limit, \( \omega \rightarrow 0 \), wave number should lead to zero which means that the plasma gets a steady state.

Using the Lorentz transformed conductivity tensor of the plasma components and the total dielectric permittivity tensor of the plasma in the laboratory frame, we have investigated the effect of thermal motion of charged particles in the filamentation of a nonrelativistic current-driven plasma in the diffusion and ion-acoustic frequency regions. A threshold for instability development is obtained. Furthermore, we have shown that close to the threshold, \( k \gg k_0 \), the current layer will be subdivided into separate current filaments with a period of the order of \( \gamma_c \) and the establishment time of the order of \( 2 \nu_1 / u \omega_{\gamma_c}^2 \).

References

