ELECTRON BEAM RELAXATION IN WEAKLY IONIZED GAS

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ABSTRACT

This paper concerns the theoretical treatment of the problem of electron beam relaxation in weakly ionized molecular gas on the base of modified quasilinear equations. The relaxation length is evaluated in regymes which are interesting for the plasmachemical applications of the stationary plasma-beam discharge.

Stationary plasma-beam discharge (SPBD) was proposed for plasma chemical applications in /1/. The sequential investigations, theoretical and experimental as well, had shown the SPBD ability of dealing practically with every kind of gas in wide range of pressures ($10^{-4} \div 10$ torr) and powers of the system, and achievinga high degree of conversion of initial components (practically as full as possible) with high efficiency. The utilization of SPBD in plasmachemistry imposes a number of restrictions on parameters of electron beam and plasma created by the first one. The most essential of these concerning the main features of beam-plasma interaction can be written as follows:

(1) $$w_L > w_C$$

(w_L - designates the electron beam pomer dissipating during the generation of Langmuir oscillations (LO); w_C - the same but for collisions of beam electrons with the particles of weakly ionized molecular gas (WIMG)). Another restrictions are imposed by the concrete chemical processes e.g.: it is well known that a number of the chemical reactions can be effectively accelerated via vibrational excitation (VE) of molecules, and since the cross-sections of VE via electron impact have the spikes near the energy of $1 \div 2$ eV the average electron energy must be the same order of magnitude to achieve high efficiency, and the main part of the energy input must be spent on the heating of plasma electrons having the energy near the average. The last condition imposes some restrictions on the heating mechanism of electrons. Actually, it is known that effective electron heating occurs during the so-called modulation instability (MI) of LO, however, in this process energy is absorbed by the "tail" of energy distribution of electrons /2/, and efficiency of VE is much less than unity. On the other hand the uniform heating is possible in the form of Joule dissipation of LO. The last case takes place when:

(2) $$w_L > w_M$$

(w_M - dissipation of LO energy in the process of MI).

To assess the range of parameters where unequalities (1,2) take place, let us analyse the electron beam relaxation process assuming (1,2) fulfilled. The relaxation process is usually considered in plasmas with fixed concentration n_e, however, in the case of WIMG plasma concentration ultimately depends upon energy density of LO W_o, and thus the beam instability increment dependence $\gamma \sim 1/\sqrt{n_e}$ (" $\sim$ " - sign of proportionality) upon W_o can effect the relaxation process /3/. The kinetic equation for electrons with Joule heating is the following /4/:

$$(3) \quad \frac{df_e}{dt} = \frac{W_o}{n_e} D_\varepsilon (f_e) + St\,(f_e) + D_e(f_e)$$

(f_e - energy distribution function of electrons; the first term in r.h.s. of (3) corresponds to electron heating via the scattering on heavy particles of WIMG; the second - elastic and inelastic interaction of electrons with WIMG including Coulomb one; the third - life-limiting processes). One can obtain the main features of electron density dependence upon W_o without direct solution of (3). Actually, let us consider low degree of ionization so that one can neglect the Coulomb interaction, and life-time of electrons τ undependable on n_e. Thus the only parameter of (3) is the ratio W_o/n_e which can be evaluated from the balanca equation of number of electrons:

$$(4) \qquad K_I(f_e(W_o/n_e))\, N_o = 1/\tau$$

(K_I - ionization constant; N_o - concentration of neutrals). Thus distribution function of electrons $f_e(\,\mathcal{E}\,)$, their average energy $\frac{3}{2}T_e$ depend mainly upon τ but not W_o, and n_e is proportional to W_o. The factor between n_e and W_o can be obtained from the equation of electron energy balance which in the case when the main part of electron energy is spent on VE of molecules and the deexcitation process is taken into acount has the following form:

$$(5') \quad W_o\,(\nu_{en} + \nu_{ei}) = \hbar\omega \cdot \nu_{ev}\, n_e(1 - \exp(-\hbar\omega(\tfrac{1}{T_v} - \tfrac{1}{T_e})))$$

$$(5'') \quad \hbar\omega\ \nu_{ev}\, n_e\,(1 - \exp(-\hbar\omega\,(\tfrac{1}{T_v} - \tfrac{1}{T_e}))) = T_v\, N_o/\tau$$

(ν_{en}, ν_{ei} - frequencies of electron-neutral and -ion impacts respectively; ν_{ev} - frequency of excitation of vib. levels). System (5',5'') is derived in assumption of Boltzmann population of molecular vib. levels with temperature T_v, and that the excitation frequencies for different levels are equal one another. When τ is quite small one can obtain from (5',5'') the following dependence:

$$(6) \quad n_e = W_o \cdot \nu_{en}/(\hbar\omega\ \nu_{ev}\,(1 - W_o/W_{max}))$$

($W_{max} = \hbar\omega \cdot n_b \; \nu_{ev} / \nu^b_{ei}$; $\nu^b_{ei} = \nu_{ei} \, n_b/n_e$; n_b -initial electron beam density). Relation (6) is consistent with the results of detailed computer analysis /5/. If

$\tau \gtrsim ((\frac{\hbar\omega}{T_e})^2 \frac{n_e}{N_o} \nu_{ev})^{-1}$ one can obtain from (5',5'') $T_v \simeq T_e$

and (7) $\quad W_o \, (\nu_{en} + \nu_{ei}) = T_e \, N_o / \tau$.

Let us now consider the system of quasilinear equations which gives the unidimensional description of the electron beam relaxation process:

$$(8) \quad v_g^k \frac{dW_k}{dx} - \frac{d\omega_{pe}}{dx} \frac{dW_k}{dk} = (2\gamma_k - (\nu_{en} + \nu_{ei}))\, W_k \; ;$$

$$(9) \quad v \frac{df_b}{dx} = 4\pi^2 (e/m)^2 \frac{d}{dv} \left(\frac{W_k}{v} \frac{df_b}{dv} \right) \; ; \quad kv = \omega_{pe} \; ;$$

$$(10) \qquad W_o = \int W_k \, dk = W_o(n_e)$$

($f_b(v)$ - distribution function of electron beam in velocity space; W_k - spectral energy density of LO; v_g^k - group velocity of LO; $\omega_{pe} = (4\pi n_e e^2/m)^{1/2}$- Langmuir frequency;

$\gamma_k = \frac{1}{2}\pi\omega_{pe} \, (n_b/n_e) \, v^2 \, (df_b/dv) \sim 1/\sqrt{n_e}$ - increment of plasma-beam instability). Function $f_b(v)$ satisfies an equation of continuity: $\int v \, f_b(v) \, dv = v_o$ (v_o is the initial velocity of the beam; note, that n_b thus designates initial concentration of electrons of the beam). Equation (10) takes into acount the relation between n_e and W_o obtained above. On the first stage of relaxation $2\gamma_k \gg (\nu_{en} + \nu_{ei})$, and the main term which stabilizes the instability in (8) is a repumping of LO along the spectrum owing to unhomogeneity of electron density n_e. Thus neglecting the term $v_g(dW_k/dx)$ in (8) one can obtain the following approximate equations:

$$(11) \quad \frac{d}{dx} \left(\frac{n_e}{n_b} \right)^{1/2} = \frac{\pi}{\Lambda} \frac{\omega_{pb}}{v_o} \left(\frac{v_o}{\Delta v} \right)$$

$$(12) \quad \frac{d}{dx} \left(\frac{\Delta v}{v_o} \right) = \frac{1}{2} \pi \left(\frac{n_b}{n_c} \right)^{1/2} \left(\frac{v_o}{\Delta v} \right)^2 \frac{\omega_{pb}}{v_o} \frac{W_o}{m v_o^2/2 \cdot n_b}$$

($\omega_{pb} = (4\pi n_b e^2/m)^{1/2}$; Λ - Coulomb logarithm; Δv - the spread of beam electrons in velocity space owing to relaxation process). Equations (11,12) are quite valid until the length x when 2γ becomes approximately equal $(\nu_{en} + \nu_{ei})$ or $\Delta v/v_o \gtrsim 1$, i.e. the relaxation process becomes essentially three-dimensional. Using the integral of (11,12)

$$\left(\frac{v(x)}{v_o}\right)^2 - \frac{\Lambda}{n_b m v_o^2/2}\int\frac{W(n)}{n}\,dn = \mathrm{Const}$$ one can assess the length of the first stage of relaxation which is described by the system (11,12) for each of the typical regions of beam-gas parameters E, G and F (fig. 1):

$$(13)\quad L_E \simeq \frac{1}{2A_n}\left(\frac{1}{E_o}\,\frac{\mathcal{V}_{max}}{\mathcal{V}_b}\right)^{1/2}\ ;\quad E_o = \frac{1}{2}\Lambda\cdot\mathcal{V}_{max}\ ;$$

$$(14)\quad L_G \simeq L_E\cdot E_o\cdot\Gamma_o^{2/3}\ ;\qquad \Gamma_o = \pi\frac{\omega_{pb}}{\nu_{en}}\left(\frac{\mathcal{V}_b}{\mathcal{V}_{max}}\right)^{1/2}\frac{1}{E_o}$$

$$(15)\quad L_F \simeq 2\,L_G\cdot\Gamma_o^{-1/3}\ ;\quad A_n = \frac{\pi\,\omega_{pb}}{\Lambda\,v_o};\quad \frac{\mathcal{V}_{max}}{\mathcal{V}_b} = \frac{\nu^b_{ei}}{\nu_{ev}}\ .$$

Let us consider the second stage of beam relaxation when one can neglect the term in (8) which corresponds to spectral re-pumping of LO/3/. There are four typical regions of parameters of the system "gas-beam" on this stage (fig. 2): E, I, N and IN. Region E which is one where essentially three-dimensional consideration is required - is not interesting for plasma-chemical applications as the evaluation shows. The region designated as I is characterized as follows: $2\gamma \simeq \nu_{ei} \gg \nu_{en}$, while N - $2\gamma \simeq \nu_{en} \gg \nu_{ei}$. In the region IN it is initially fulfilled $2\gamma \simeq \nu_{ei} \gg \nu_{en}$ and finally $2\gamma \simeq \nu_{en} \gg \nu_{ei}$. As the evaluation shows the relaxation length for the region IN is mainly defined by the last regyme. Thus solution of the system (8-10) gives the following evaluations for the relaxation lengths:

$$(16)\quad L_{IN} \simeq L_N \simeq \frac{1}{6\pi^2}\,\frac{mv_o^2/2}{\hbar\omega}\,\frac{\nu_{en}}{\nu_{ev}}\,\frac{\nu_{en}v_o}{\omega^2_{pb}}$$

$$(17)\quad L_I \simeq \frac{(2\pi)^{1/3}}{3\pi}\,\frac{mv_o^2/2}{\hbar\omega}\,\frac{v_o}{\nu_{en}}\left(\frac{\nu^b_{ei}}{\omega_{pb}}\right)^{2/3}$$

Let us compare the relaxation lengths with ones of the first stage. It is easy to show that

$$\frac{L_N}{L_G} = \frac{1}{(\Gamma_o E_o^{3/2})^{5/3}} > 1\ ;\qquad \frac{L_I}{L_F} \gtrsim \frac{1}{E_o^{7/6}} > 1\ .$$

The last unequalities show that the total relaxation length is defined by the second stage. In above consideration we have assumed that the time spent by the molecules in the discharge zone is quite small so that $T_v \ll T_e$. Let us now assess the influence of population of vibrational levels on electron beam relaxation. Consider the second stage of relaxation assuming for the sake of simplicity that $\nu_{en} \gg \nu_{ei}$. If unequality

$$(18) \qquad \tau < \left(4\pi^2 \left(\frac{\hbar\omega}{T_e}\right)^2 \cdot \nu_{ev} \frac{n_b}{N_o} \left(\frac{\omega_{pb}}{\nu_{en}}\right)^2\right)^{-1}$$

is fulfilled one can obtain $T_v \ll T_e$ and the following equation for v_m - co-ordinate of the lower front of the distribution function $f_b(v)$ ($f_b(v_m)=0$; $v_m(x) < v_o$):

$$(19) \qquad \frac{dv_m}{dx} = -6\pi^2 \frac{\hbar\omega}{mv_o^2/2} \frac{\nu_{ev}}{\nu_{en}} \frac{\omega_{pb}^2}{\nu_{en} v_o} \frac{v_m^2}{(v_o-v_m)^4} \frac{2v_o^6}{(v_o+v_m)}$$

If (18) is not fulfilled one can obtain $T_v \simeq T_e$ and

$$(20) \qquad W_o = N_o T_e / (\tau \cdot \nu_{en}) .$$

Equation (19) transforms in the last case to

$$(21) \qquad \frac{dv_m}{dx} = -\frac{3}{2} \frac{T_e}{mv_o^2/2} \frac{N_o}{n_b} \frac{2v_o}{v_m+v_o} \frac{1}{\tau} .$$

Thus a high population of vibrational levels leads to stabilization of W_o given by (20) and an increase of relaxation length. Note that in this case energy input per unit volume of WIMG will be conatant and independent upon beam parameters. Let us now determine the region of parameters of the problem where unequalities (1,2) are fulfilled. Unequality (1) is adequate to the following: the relaxation length is to be smaller than the quantity

$$L_C = \frac{mv_o^2}{2\mathcal{E}_b} \frac{v_o}{\nu_{bn}}$$ which is the order of magnitude of

classical relaxation length on binary collisions ($\mathcal{E}_b$ - average energy loss per one impact; ν_{bn} - frequency of binary collisions of beam electrons with gas particles). The detailed analysis shows that for all typical regions of parameters unequality (1) can be transformed to

$$(22) \qquad \frac{(\nu_{en} \nu_{ei}^b / \omega_{pb}^2)^{1/2}}{6\pi} \frac{\mathcal{E}_b}{\hbar\omega} \frac{\nu_{bn}}{\nu_{ev}} < 1 .$$

Consider unequality (2) now. The more severe nonlinear process which leads to energy dissipation in collisional plasma is a modulation instability (MI) as mentioned above. The last one can be neglected if

$$(23) \qquad \gamma_M \simeq \omega_{pe} \left(\frac{1}{3} \frac{m}{M} \frac{W_o}{n_e T_e}\right)^{1/2} < \nu_{en} + \nu_{ei} \quad /6/.$$

(M - mass of an ion)

Substituting the typical values of W_0 and n_e to (23) for each region of parameters one can obtain (the intermediate derivations are omitted) that the last unequality is fulfilled for gas concentrations more than 10^{15} cm^{-3} and moderate another parameters of beam and plasma.

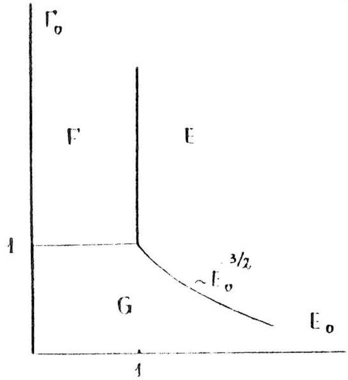

Fig. 1

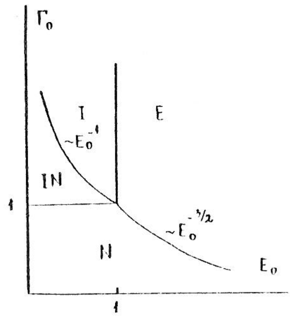

Fig. 2

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