ELECTRON SWARM TRANSPORT IN HIGH FREQUENCY DISCHARGE PLASMA

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ABSTRACT

Electron swarm transport in radio-frequency fields in gases is analyzed from the Boltzmann equation considering the harmonic components of the velocity distribution. Discussion is, particularly, focused both on the behavior of the harmonics of the distributions to the energy and on the time dependence of the swarm parameters as the functions of $\omega$ and $E/N$. Numerical analysis has been made for the electron swarm in argon over $10^6 \leq \omega \leq 10^{11}$ s$^{-1}$.

1. INTRODUCTION

There has been an extensive use of the radio-frequency (rf) discharges for plasma processing such as plasma deposition and etching in microelectronic manufacturing. A typical plasma reactor consists of the parallel plate rf discharges. There will exist two kinds of transports of electrons in the reactor. Namely, one is the beam-like transport between the electrode and the sheath. The other is the swarm transport in the bulk of the plasma.

Theories early developed by Holstein(1946) and Margenau(1948) give the satisfactory explanations of the DC properties of the electrons in the plasma bulk. The isotropic part of the distribution of electrons moving through the medium under the influence of the rf field $E(t) = E \cos(\omega t)$ behaves as if the effective DC field

$$E_{\text{eff}} = \frac{1}{\sqrt{1 + (\omega/\nu(\epsilon))^2}} E$$

is applied. $\nu(\epsilon)$ is the total collision frequency of the electron with the energy $\epsilon$.

Recent measurements by Rosny et al(1983) and Gottscho et al(1984) on the optical emissions in the rf glow discharges show, however, that the emission intensities are strongly modulated in time. These new-observed phenomena indicate the needs for further theoretical study of the rf glow discharge plasma.

This paper describes the electron transport in the rf discharge plasma-bulk, especially, for the harmonic components from the solution of the Boltzmann equation.
2. Theory

We deal with the rf discharge plasma driven by a uniform and time-varying rf fields

\[ E(t) = E \exp(j \omega t) \]  

where \( \omega \) is the rf angular frequency and \( t \) is the time. In the bulk of the plasma, the reduced field strength \( E/N \) is, in practice, small and the velocity distribution of electrons is nearly isotropic in velocity space.

The electron density will be a spatially relaxed and homogeneous in the axial direction parallel to the field. In these circumstances, the velocity distribution of the electron swarm \( g(v,t) \) obeys the Boltzmann equation

\[ \frac{\partial}{\partial t} g(v,t) + \nabla_v \cdot \left( \frac{eE(t)}{m} g(v,t) \right) = J(g) \]  

where \( e \) and \( m \) are the charge and the mass of the electron, \( v \) is the velocity, and \( J \) denotes the collision term. The velocity distribution is expressed both by the Legendre polynomial in velocity space and by the Fourier expansion in time.

\[ g(v,t) = \sum_{\ell} \sum_{\kappa} g^{\ell}_{\kappa}(v) P(\cos \theta) \exp(j \omega t). \]  

Here, \( \ell \)th Legendre and \( \kappa \)th Fourier component is denoted as \( g^{\ell}_{\kappa}(v) \) by the subscript \( \ell \) and the superscript \( \kappa \). \( P(\cos \theta) \) is Legendre polynomials and is given with \( \theta = \cos^{-1}(v \cdot E/|E|) \). In purely harmonic fields in time, the isotropic part of the distribution \( g^{0}_{0}(v) \) has only even harmonics, while the first-anisotropic \( g^{1}_{0}(v) \) has odd one (Margenau and Hartman 1948).

Physically acceptable truncation of \( g^{\ell}_{\kappa}(v) \) is given by \( \ell + \kappa = 2s, s = 1,2, \ldots \)

The lowest-order approximation is given \( \ell = 0, \kappa = 2 \) in the case where a periodic time variation appears in the electron density. Therefore, consideration will be paid to the components of the distribution, \( g^{0}_{0}(v), g^{0}_{1}(v) \) and \( g^{0}_{2}(v) \). Here, \( g^{0}_{0}(v) \) and \( g^{0}_{1}(v) \) are complex and the real part of them are taken, while \( g^{0}_{0}(v) \) and \( g^{0}_{2}(v) \) are real. A set of scalar and vector differential equations is derived from equation (3).

\[ \frac{1}{3v^2} \frac{\partial}{\partial v} \left[ \frac{v^2}{2} \operatorname{Re} \left( \frac{eE}{m} g^{1}_{1}(v)^* \right) \right] = J_{e1}(g^{0}_{0}(v)) + J_{inel}(g^{0}_{2}(v)). \]

\[ j2w g^{0}_{1}(v) + \frac{1}{3v^2} \frac{\partial}{\partial v} \left[ \frac{v^2}{2} \frac{eE}{m} g^{1}_{1}(v) \right] = J_{e1}(g^{0}_{0}(v)) + J_{inel}(g^{0}_{2}(v)). \]

\[ jw g^{1}_{1}(v) + \frac{3}{2v} \frac{\partial}{\partial v} \left[ \frac{v^2}{2} g^{0}_{0}(v) \right] + \frac{eE}{m} \frac{\partial}{\partial v} \left[ \frac{v^2}{2} g^{0}_{2}(v) \right] = - \frac{2}{3} \frac{\partial}{\partial v} \left[ \frac{v^2}{2} g^{0}_{0}(v) \right] \]

\[ \frac{2v}{3} \frac{\partial}{\partial v} \left[ \frac{1}{2v} \operatorname{Re} \left( \frac{eE}{m} g^{1}_{1}(v)^* \right) \right] = - \nu\nu_{\kappa m}^{2}(v) g^{0}_{2}(v). \]

The collision terms are expressed as

\[ J_{e1}(g^{0}_{0}(v)) = \frac{m}{M} \frac{1}{v^2} \frac{\partial}{\partial v} \left[ \frac{v^2}{2} \nu_{\kappa}^{\nu_{\kappa}}(v) g^{0}_{0}(v) \right], \]

\[ J_{inel}(g^{0}_{0}(v)) = \frac{1}{v^2} \left[ \nu_{\kappa}^{\nu_{\kappa}}(v) g^{0}_{0}(v) \right] \frac{v^2}{\nu_{\kappa}^{\nu_{\kappa}}(v) + v^2} \]

\[ + \frac{1}{v^2} \left[ \nu_{\kappa}^{\nu_{\kappa}}(v) g^{0}_{0}(v) \right] \frac{v^2}{\nu_{\kappa}^{\nu_{\kappa}}(v) + v^2} \]

\[ + \frac{1}{v^2} \left[ \nu_{\kappa}^{\nu_{\kappa}}(v) g^{0}_{0}(v) \right] \frac{v^2}{\nu_{\kappa}^{\nu_{\kappa}}(v) + v^2} \]

\[ + \frac{1}{v^2} \left[ \nu_{\kappa}^{\nu_{\kappa}}(v) g^{0}_{0}(v) \right] \frac{v^2}{\nu_{\kappa}^{\nu_{\kappa}}(v) + v^2} \]

Here, \( \nu_{\kappa}^{\nu_{\kappa}}(v) = \nu_{\kappa}^{\nu_{\kappa}}(v) + \nu_{\kappa}^{\nu_{\kappa}}(v) + \nu_{\kappa}^{\nu_{\kappa}}(v) \), and \( \nu_{\kappa}^{\nu_{\kappa}}(v) = \nu_{\kappa}^{\nu_{\kappa}}(v) + \nu_{\kappa}^{\nu_{\kappa}}(v) + \nu_{\kappa}^{\nu_{\kappa}}(v) \). The momentum transfer and the viscosity cross sections of the elastic scattering. \( Q_{\kappa}(v) \) is the excitation cross section with the threshold energy \( E_{i} = m v^{2}/2 \), and \( Q_{i}(v) \) the ionization cross section with ionization energy \( E_{i} = m v^{2}/2 \).
It is convenient to define the energy distribution \( f^K_{\ell}(\varepsilon) \) according to the relation

\[
f^K_{\ell}(\varepsilon) = (4\pi/n)\nu g^K_{\ell}(\nu), \quad \varepsilon = mv^2/2.
\] (12)

The macroscopic parameters, which are derived from the time-dependent distributions, are expressed as follows. The electron density

\[ \text{Re}[n(t)] = \text{Re}\left[ \Sigma \int f^0_{\ell}(\varepsilon) \exp(j\omega t) d\varepsilon \right] = n^0 + \text{Re}[n^2(t)], \] (13)

where, \( n^0 = \int f^0_{\ell}(\varepsilon) d\varepsilon \) and \( n^2(t) = \int f^2_{\ell}(\varepsilon) \exp(j2\omega t) d\varepsilon = |n^2| \exp(j2\omega t + \phi^2) \).

The average of any scalar quantity \( H(\varepsilon, t) \) is

\[
\text{Re}[\overline{H}(t)] = \text{Re}\left[ \Sigma [H(\varepsilon, t)f^0_{\ell}(\varepsilon) \exp(j\omega t)] d\varepsilon \right] / \text{Re}[n(t)]
\]

\[ \equiv \overline{H}^0 \{ 1 + \frac{1}{2} \left( |n^2| / n^0 \right)^2(1 - \left| H^2 \right| / \overline{H}^0) \}
\]

\[- \cos(2\omega t + \phi) \left| n^2 \right| n^0 \left( 1 - \left| H^2 \right| / \overline{H}^0 \right) \]

\[- \cos(4\omega t + 2\phi) \left( \frac{1}{2} \left( |n^2| / n^0 \right)^2 \right) \}
\]

(14)

where, \( \overline{H}^0 = \int [H(\varepsilon, t)f^0_{\ell}(\varepsilon) d\varepsilon / n^0 \), and \( \overline{H}^2(t) = \text{Re}[\Sigma [H(\varepsilon, t)f^2_{\ell}(\varepsilon) \exp(j2\omega t)] / \text{Re}[n^2(t)] \). Each of the distribution is numerically calculated from the simultaneous differential equations (5)-(8), including the six unknown functions \( f^0_0(\varepsilon) \), \( \text{Re}[f^0_0(\varepsilon)] \), \( \text{Im}[f^0_0(\varepsilon)] \), \( \text{Re}[f^2_0(\varepsilon)] \), \( \text{Im}[f^2_0(\varepsilon)] \) and \( f^2_0(\varepsilon) \).

### 3. RESULTS & DISCUSSIONS

![Graph](image.png)

**Figure 1.** Typical features of each component of the energy distribution in argon in rf fields.

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The energy distribution and the swarm parameters of electrons are calculated in argon over the range $10^6 < \omega < 10^{11} s^{-1}$ at $E/N \approx 60$ Td and $T_e = 273$ K as an example. A typical characteristic of each component of the distributions is shown in figure 1 for $\omega=10^8$ s$^{-1}$ and $E/N=50$ Td. The DC component of the isotropic part of the distribution, $f_0^0(\varepsilon)$, is normalized to unity. $f_0^0(\varepsilon)$ shows the conventional dependence on energy, except the distorting germ in the vicinity of zero energy. Namely, $f_0^0(\varepsilon)$ has the property in the effective DC field given by the equation (1). Two-term theory approximately gives the collisional relaxation time $\tau_0(\varepsilon)$ of the isotropic distribution $f_0^0(\varepsilon, \varepsilon)$ in the rf field.

$$\tau_0(\varepsilon) = \frac{2e}{m} v_m(\varepsilon) + \Sigma v_j(\varepsilon) + \nu_{\perp}(\varepsilon)$$

(15)

where $V(\varepsilon)$ is the collision frequency equal to $NQ(\varepsilon)\sqrt{2e/m_e}$. The electron with the energy $\varepsilon<\varepsilon_j$ will be relaxed in time only by the elastic collisions, and the condition $\tau_0(\varepsilon) > \omega$ is satisfied because of the small mass ratio $m/M$ between the electron and the molecule. In this case, $f_0^0(\varepsilon, \varepsilon)$ behaves independently of time even in the rf field as shown in figure 1, i.e.

$$f_0^0(\varepsilon) = f_0^0(\varepsilon).$$

At high energy-electron with $\varepsilon > \varepsilon_j$, the relaxation will be governed mainly by the inelastic collisions. Then, $\tau_0(\varepsilon) - \omega$ will be realized. As the result, very interesting feature appears in the harmonic term $f_0^2(\varepsilon)$. The magnitude of $|f_0^2(\varepsilon)|$ at high energy is comparable with $f_0^0(\varepsilon)$ and is never neglected. These region with the strong time-modulation extend gradually to lower energy as decreasing of $\omega$. This means that the excitation or ionization frequency will show the strong time-dependence with a period of $\pi/\omega$, compared with the quantity estimated all over the energy of $f_0^0(\varepsilon, \varepsilon)$. On the other, the relaxation time of $f_0^1(\varepsilon, \varepsilon)$ will be estimated as

$$\tau_1(\varepsilon) = \frac{V_e(\varepsilon)}{m} v_m(\varepsilon) + \Sigma v_j(\varepsilon) + \nu_{\perp}(\varepsilon).$$

(16)

Since $\tau_1(\varepsilon) < \omega$ is satisfied $f_0^1(\varepsilon, \varepsilon)$ is shown as $|f_0^1(\varepsilon)| \cos(j\omega t)$ without DC component.

Next, the discussion will be given for the phase-shift of $f_0^1(\varepsilon)$ and $f_0^2(\varepsilon)$, i.e. $\phi^1(\varepsilon)$ and $\phi^2(\varepsilon)$. $f_0^1(\varepsilon, \varepsilon)$ is analytically expressed from the equation (7) as

$$f_0^1(\varepsilon, \varepsilon) = \frac{\sqrt{2e/m} \omega E}{(1+\omega E)} \left\{ \frac{1}{1+\omega E} e^{i\phi^0(\varepsilon)} \right\} \frac{1}{2\sqrt{2e/m \omega E}} \frac{d}{dE} \phi^0(\varepsilon)$$

(17)

If $\omega << \Sigma v(\varepsilon)$ is satisfied, then $\phi^1(\varepsilon)$ is close to zero. On the other, $\phi^1(\varepsilon)$ is $\tan^{-1}(\omega/\Sigma v(\varepsilon))$ and tends to -$\pi/2$, if $\omega \gg \Sigma v(\varepsilon)$. The phase-shift of $f_0^1(\varepsilon, \varepsilon)$, $\phi^1(\varepsilon)$ is somewhat complicated than $\phi^0(\varepsilon)$. At the energy $\varepsilon > \varepsilon_j$,

$$f_0^2(\varepsilon) >> f_0^0(\varepsilon, \varepsilon)$$

is realized as $|f_0^0(\varepsilon)|$ decreases so rapidly. Then, $f_0^2(\varepsilon, \varepsilon)$ is approximated by

$$f_0^2(\varepsilon, \varepsilon) = \frac{-1+i(2\omega/\Sigma v(\varepsilon))}{(1+(2\omega/\Sigma v(\varepsilon))^2)} \frac{eE}{6} \frac{d}{dE} \frac{f_0^1(\varepsilon)^2}{m} \exp(i2\omega t).$$

(18)

As the result, $\phi^2(\varepsilon) = \tan^{-1}(2\omega/\Sigma v(\varepsilon))$ tends to zero, if $\omega < \Sigma v(\varepsilon)$. Figure 2 shows the phase-shift as a function of energy at $\omega=10^8$ s$^{-1}$.

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Figure 3 shows the DC-part of the isotropic distribution $f^0_0(\varepsilon)$ as a parameter of angular frequency $\omega$ at $E/N$ of 60 Td. All of the functions are normalized to unity. The shape of $f^0_0(\varepsilon)$ scarcely changes at $\omega < 5 \cdot 10^8 \text{s}^{-1}$ over the range of $\omega$ investigated, since the condition $\omega < \Sigma \nu_j(\varepsilon)$ holds. In the range $\omega > \Sigma \nu_j(\varepsilon)$, the shape changes so rapidly as increasing of $\omega$. And, ultimately, Maxwellian will be realized with the effective temperature

$$T_{\text{eff}} = T \left(1 + \frac{\partial M}{\partial E} \frac{E}{\omega} \right)^2$$

(Margenau & Hartman 1948).

We investigate the influence of the 2nd harmonic component of the isotropic distribution on the swarm parameters. The time dependence of the total excitation collision frequency, $\Sigma \nu_j(t)$ is compared with that of the mean energy $\bar{\varepsilon}(t)$ in figure 4 at $E/N=50$ Td and $\omega=10^8 \text{s}^{-1}$.

![Figure 3. Dependence of $f^0_0(\varepsilon)$ on the angular frequency $\omega$ at $E/N$ of 60 Td.](image)

![Figure 4. Time dependence of the swarm parameters, $\bar{\varepsilon}(t)$, $\Sigma \nu_j(t)$ and $n(t)$ derived from figure 1, at $\omega = 10^8 \text{s}^{-1}$ and $E/N = 50$ Td.](image)

$\Sigma \nu_j(t)$ is calculated from the equation (14) replacing $N(\varepsilon)$ with $2NQ_j(\varepsilon)/2E\omega$. The threshold of the excitation appears at above $\varepsilon_j=11.6$ eV in argon, and $|f^0_0(\varepsilon)|$ has a comparable magnitude with $f^0_0$ at $\varepsilon > \varepsilon_j$ as mentioned above. $\Sigma \nu_j(t)$ is, therefore, modulated enough in comparison with $\bar{\varepsilon}(t)$ calculating from $\int f^0_0(\varepsilon, t) \, d\varepsilon$ over the whole range of energy. It should be noticed that the physical situation undergoes a complete change in molecular gases because the molecule has some vibrational excitations at very low energies. That is, the modulation of $f^0_0(\varepsilon, t)$ extends to the lower energy which corresponds to the vibrational threshold of the molecule. In figure 4, the maximum and the minimum of the parameters are exhibited by $\dagger \dagger$. Attention will be paid to the difference of the phase-delay between $\bar{\varepsilon}(t)$ and $\Sigma \nu_j(t)$.

The ratio of the 2nd harmonic component to DC of the electron density is shown in figure 5 as a function of $\omega$. Rapidly decreasing characteristic appears.

Figure 6 shows the time characteristics of the total excitation frequency $\Sigma \nu_j(t)$ as a parameter of $\omega$ at $E/N$ of 50 Td. The energy range, satisfying $|f^0_0(\varepsilon)|/f^0_0(\varepsilon) \sim 1$, shifts gradually to high energy as increasing of $\omega$ at $\omega < \Sigma \nu_j(\varepsilon)$. The DC-component is, therefore, unchanged and the modulation amplitude is reduced to zero as is shown in figure 6(a), (b), (c), (d). The higher order-even harmonics will appear in addition to the 2nd harmonics, when the ratio of $|n^2|$ to $n^0$ grows with the decrease of $\omega$, as is expected.
theoretically from the equation (14). This feature is recognized in figure 6(a). The phenomena that the rising of $\Sigma v_j(t)$ is faster than the fall will be understood from the cross-sectional behavior to the energy. The phase delay grows with the increase of $\omega$.

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REFERENCES

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