Numerical Simulation of Magnetically Rotating Arc Plasma

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Abstract: Magnetically rotating arc plasma exhibits typically dispersed uniform arc column as well as diffusive cathode root and diffusive anode root. In this paper a 2D model of unified arc column with electrodes is used to simulate magnetically rotating arc plasma. The numerical simulation results show that the configuration of the cathode arc root varies with the cathode shape, gas flow, imposed axial magnetic field, arc currents and etc. Moreover the configuration of the cathode arc root affects the configuration of the arc column as well as one of the anode arc root.

Key words: magnetically rotating arc, dispersed arc plasma, arc plasma configuration; diffusive arc root

1. Introduction

The arc plasma is of special application in industry, for its high temperature and enthalpy, such as in cutting, welding, surface processing, metallurgy, heat-resisting material production and etc. Being highly concentrated energy and smaller volume, the arc plasma is difficult in large scale industry application, especially in material production, for its undesirable products uniform state. Nevertheless, some special configurations of the cathode arc root, such as split multi-spots, diffusive spot and diffusive doughnut, and etc., have not been obtained in those simulations.

It may be inferred that the configuration of cathode arc root would affect the arc column configuration, so as to the arc anode root configuration. While the configuration of the cathode arc root may be affected by its adjacent plasma, which may be affected by plasma flow and heat transfer, so as to be affected by the conditions of domain boundary and technology.

In this paper, a complete rotational symmetric dispersed arc plasma resulted from magnetically rotating arc is simulated by using a 2D LTE MHD simplified model of the arc plasma coupled with the cathode and the anode under various technical and boundary conditions.

2. Model and computation domain

In the arc column region, argon plasma is assumed in LTE (local thermodynamic equilibrium) and LCE (local chemical equilibrium), turbulent flow state, and MHD equations are applied. In near cathode region (0-0.1mm away from cathode surface), LCE is not fulfilled and the density of electron numbers as a result of solving electron continuity equation is used to calculate the so-called effective conductivity. Details of this method from J. J. Lowke and R Morrow’s unified theory of arc and electrodes can be found in reference [9][10].

Recently, by applying J. J. Lowke’s method, B Bai focused on the simulation of magnetically dispersed arc using plasma-coupling-cathode model (PCCM)[11][12]. The work in this paper is done mostly following the way of reference[6][12].

The Governing equations are as follows:
Continuity equation:

\[ \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{\partial}{\partial z} (\rho v_z) = 0 \]  

(1)

Momentum conservation equations

\[ (F_\theta = -j_r B_z, F_r = -j_z B_\theta, F_z = j_r B_\theta): \]

\[ \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r^2) + \frac{\partial}{\partial z} (\rho v_z v_r) = - \frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left( 2 r \Gamma_u \frac{\partial v_r}{\partial r} \right) + \frac{\partial}{\partial z} \left( \Gamma_u \frac{\partial v_r}{\partial z} + \Gamma_u \frac{\partial v_z}{\partial r} \right) - 2 r \frac{\Gamma_u v_r}{r^2} + F_r \]  

(2)

\[ \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_z v_r) + \frac{\partial}{\partial z} (\rho v_z^2) = - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z} \left( 2 r \Gamma_u \frac{\partial v_z}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \Gamma_u \frac{\partial v_z}{\partial r} + \Gamma_u \frac{\partial v_z}{\partial r} \right) + F_z \]  

(3)

\[ \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_z v_\theta) + \frac{\partial}{\partial z} (\rho v_z v_\theta) = \frac{\partial}{\partial z} \left( \Gamma_u \frac{\partial v_\theta}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_\theta}{\partial r} - v_\theta \right) + \frac{2}{r} \Gamma_u \left( \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right) - p \frac{v_z v_\theta}{r} + F_\theta \]

(4)

Energy conservation equation:

\[ \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_z h) + \frac{\partial}{\partial z} (\rho v_z h) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \Gamma_u \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( \Gamma_u \frac{\partial T}{\partial z} \right) + \frac{\dot{\epsilon} + \dot{\epsilon}_f}{\sigma} + U \]  

(5)

Electric potential equation:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \sigma \frac{\partial \phi}{\partial r} \right) + \frac{\partial}{\partial z} \left( \sigma \frac{\partial \phi}{\partial z} \right) = 0 \]  

(6)

Magnetic potential equations:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \sigma \frac{\partial A_z}{\partial r} \right) + \frac{\partial}{\partial z} \left( \sigma \frac{\partial A_z}{\partial z} \right) = - \mu_0 j_z \]  

(7)

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \sigma \frac{\partial A_r}{\partial r} \right) + \frac{\partial}{\partial z} \left( \sigma \frac{\partial A_r}{\partial z} \right) = - \mu_0 j_r + \frac{\partial \phi}{\partial r} \]  

(8)

\[ B_\theta = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \]

(9)

RNG \( K - \epsilon \) equations:

\[ \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_z K) + \frac{\partial}{\partial z} (\rho v_z K) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu_{eff} \frac{\partial K}{\partial r} \right) + \frac{\partial}{\partial z} \left( \mu_{eff} \frac{\partial K}{\partial z} \right) + G - \rho \epsilon \]

(10)

\[ \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_z \epsilon) + \frac{\partial}{\partial z} (\rho v_z \epsilon) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu_{eff} \frac{\partial \epsilon}{\partial r} \right) + \frac{\partial}{\partial z} \left( \mu_{eff} \frac{\partial \epsilon}{\partial z} \right) + \frac{\tau}{\kappa} (c_1 G - c_2 \rho \epsilon) - R_\epsilon \]

(11)

\[ G = \mu_\epsilon \left[ \left( \frac{\partial v_z}{\partial r} \right)^2 + \left( \frac{\partial v_z}{\partial z} \right)^2 + \left( \frac{\partial v_r}{\partial r} \right)^2 + \left( \frac{\partial v_r}{\partial z} \right)^2 + \left( \frac{\partial v_\theta}{\partial r} \right)^2 + \left( \frac{\partial v_\theta}{\partial z} \right)^2 \right] + \left( \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right)^2 \]

(12)
\[ R_e = \frac{\varepsilon \mu_0 n_e^2(1-n_e/n_0)}{1+\beta n_e^2} \frac{\nu^2}{\kappa}, \quad \Gamma_m = \mu + \mu_c, \quad \Gamma_T = \kappa + \frac{\mu_c}{\nu_T} \]  

In MHD equations, ohm’s law \( j = \sigma (E + v \times B) \) usually, being much smaller than \( E \), the motional electric field \( v \times B \) is neglected, thus resulting ideal MHD equations (1-9).

In cathode and anode body and anode body solid thermal conduction and electric potential equations are solved.

The simplified model of the arc coupled with the electrodes was developed by Zhu[9], [10] and Lowke[10]. The computation program with FLUENT® for LTE MHD model of the arc coupled with the electrodes has been developed by Li[5]-[7] and Bai[11], [12].

Besides the heat conduction to the cathode surface, simulation of arc plasma coupled with cathode should include the special energy transfer processes occurring at the surface, which are cooling processes due to the thermionic emission and radiation, heating process due to ion current. Heating by radiation from the plasma is neglected. For cathode, the additional energy flux \( F \)

\[ F = -\varepsilon a T_e^4 - \left| j_e \right| \Phi_e + \left| j_i \right| V_i \]  

\( \varepsilon \) is the emissivity of the surface, \( T_e \) is the surface temperature, \( a \) is the Stefan–Boltzmann constant, \( j_e \) is the electron current density, \( \Phi_e \) is the work function, it depends on the cathode material, \( j_i \) is the ion current density which is assumed to be \( j_i = |j| - |j_R| \) at the cathode surface, and \( V_i \) is the ionization potential of the plasma, where \( j = j_e - j_i \) is the current density at the surface of the cathode obtained from Poisson’s equation and \( j_R \) is the theoretical thermionic emission current density given by Richardson equation:

\[ |j_R| = A \Gamma_c^2 \exp \frac{-\Phi_e}{k_B T_e} \]  

\( A \) is the thermionic emission constant for the surface of the cathode, \( e \) is the electronic charge, \( k_B \) is Boltzmann constant. If \( |j_R| \) is greater than \( |j_i| \), we take \( j_i \) to be zero.

The temperature of the near cathode region is about the value of melting point, the non-equilibrium effects can make the near cathode region highly conduct. Therefore, we introduce the effective electric conductivity to enable Ohm’s Law in this region.

The electron continuity equation:

\[ \nabla \cdot (D_e \nabla n_e) = \frac{\gamma}{n_e^2 - n_{eq}^2} \]  

Where: \( D_e \) is the ambipolar diffusion coefficient for the local temperature given by \( \frac{2k_B T_e}{\mu_e} \), \( \mu_e \) is the ion mobility, defined by Langevin mobility. \( n_e \) is the electron density and \( \gamma = 1.1 \times 10^{-5} n_e T_e^{-1.5} \text{ cm}^2 \cdot \text{s}^{-1} \) is the electron–ion recombination coefficient, \( n_{eq} \) is the equilibrium plasma value of electron density for the local plasma temperature.

Then the effective electrical conductivity of near cathode region \( \sigma_{eff} \) is derived:

\[ \sigma_{eff} = \frac{n_e \mu_e}{n_e/(\gamma + 2 n_e \gamma) \nu_e T_T} \]  

Where \( n_0 \) is the equilibrium neutral particle density, \( \mu_e \) is the electron mobility \( \mu_e = \frac{e}{\gamma}, \nu_e \) is the collision frequency of the electron \( \nu = \frac{2 \lambda_e}{n_e}, \nu_e \) is the electron velocity defined by the local temperature, \( \lambda_e \) is the electron mean free path, \( n_T = n_0 + n_e + n_{eq} \) is the total particle density.

The computation domain is shown as in figure 1.

3. Results and discussion

Figure 2 shows that the cathode arc root by using the model of arc coupled with the cathode and transfers to the intermediate section of the cone surface from the cathode tip by using the model of given the cathode condition. The arc cathode root exhibits doughnut configuration, the spot area being larger and diffusive.
More plasma configurations under various technical and boundary conditions, including cathode arc root, arc column and anode arc root, are discussed in this paper. The mechanism of the interaction among cathode arc root, arc column and anode arc root are analyzed.

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References:

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<th>Table 1. Boundary condition</th>
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<td>E-D-C</td>
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Figure 2, temperature contour by using the model of arc coupled with cathode (coupled) and given cathode boundary conditions (fixed) (arc currents 200A, gas flow: 0.5m/s , AMF: 0.15T)