The effect of single particle charge limits on particle charge distributions in nanodusty plasmas

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Abstract: Dust particles in plasmas can hold a limited number of electrons. When a particle’s charge reaches its limit, either an extra electron is unable to attach to the particle or the particle breaks up because electrostatic repulsion on its surface overcomes surface tension. Accounting for particle charge limits, we derive an analytical expression for the stationary particle charge distribution and study the effects numerically.

Keywords: dusty plasmas, particle charging, particle charge limits, particle charge distribution

1. Introduction

A dust particle in a plasma collides with electrons and ions and therefore acquires charge. Due to the higher mobility of electrons compared to ions, particles in plasmas tend to be negatively charged. For particles smaller than about 50 nm in diameter, the discrete nature of charging as well as charge fluctuations become important. Matsoukas and Russell described stochastic charging as a one-step Markov process which could be expressed in continuous form as a Fokker-Planck differential equation where the convective and diffusive terms are functions of electron and ion currents [1]. They showed that the stationary particle charge distribution has a Gaussian form if the average particle size is not too small. However, they did not consider the fact that the maximum charge a particle can hold is limited.

The focus of the present paper is to study the effect of particle charge limits on particle charge distributions. We derive an analytical expression for stationary particle charge distributions and compare it with a Monte Carlo charging model where electron and ion currents to particles are based on the Orbital Motion Limited theory [2].

2. Particle charge limits

There is a limit to the number of charges that can coexist on a single particle. The magnitude of this limit depends on the particle’s material and size. Several phenomena that limit a particle’s charge have been studied. The classical charge limit for a spherical conducting droplet, known as the Rayleigh limit, is reached when Coulomb repulsion forces become greater than the surface tension, causing the droplet to break up [3]. The Rayleigh limit is typically unimportant for solid particles, as their surface tension is much larger than for liquids. However, when the self-generated electric field at the surface of a solid particle becomes sufficiently high, electrons are spontaneously emitted from the surface. The limiting surface electric field for electron emission is in the order of $10^7$ V cm$^{-1}$ [4]. An estimation of the particle charge limit for a semiconductor nanoparticle is discussed by Bouchoule et al. [5]. They derived an effective electron affinity for a nanoparticle, which is smaller than the bulk electron affinity due to the particle’s surface curvature and charge. In Fig. 1, we compare the three particle charge limit expressions for solid silicon nanoparticles. For very small particles, up to about 7 nm in diameter, electron field emission dominates. For larger particles the charge limit is determined by the effective electron affinity, and is given to close approximation by one charge per nm of diameter.

![Fig. 1. Negative charge limits for silicon nanoparticles.](image)

3. Analytical solution for stationary charge distribution with charge limits

Matsoukas and Russell obtained an expression for the stationary particle charge distribution that can be approximated by a normalized Gaussian distribution [1]

$$n(q) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(q-\bar{q})^2}{2\sigma^2}},$$

(1)
where $\bar{q}$ is the average charge and $\sigma^2$ is the variance of the distribution, and both $\bar{q}$ and $\sigma^2$ are functions of electron temperature and of the electron-to-ion ratios of number density, temperature, mass and sticking coefficient for collisions with nanoparticles. The Gaussian particle charge distribution given by equation (1) does not account for particle charge limits. We have derived a truncated pseudo-Gaussian for the particle charge distribution that accounts for particle charge limits, given by

$$n^*(q) = \frac{2}{1 + \text{erf} \left( \frac{q - q^*}{\sqrt{2}\sigma} \right)} \times n(q), \quad (2)$$

where $n(q)$ is the charge distribution in the absence of charge limits, given by equation (1), $\bar{q}$ is the average charge if charge limits did not exist, and $q^* = -q_{lim} - 0.5, q_{lim}$, the negative charge limit, being defined as a positive quantity [6]. In Fig. 2, we compare equation (2) with the results of numerical simulations we conducted using a Monte Carlo charging model under typical plasma conditions. Results are shown for 5 nm diameter particles either (a) without charge limit or with charge limits, (b) 14, or (c) 10. In the absence of a charge limit, the charge distribution given by equation (2) is Gaussian, as expected. With charge limits, there is good agreement between the analytical and numerical solutions.

![Fig. 2. Particle charge distributions for different charge limit magnitudes, from either equation (2) (solid lines) or Monte Carlo simulation (histograms). (a) Without charge limit; (b) $q_{lim} = 14$; (c) $q_{lim} = 10$. Plasma conditions: $n_e/n_i = 1$, $T_e = 3$ eV, $T_e/T_i = 116$, $m_e/m_i = 1.37 \times 10^{-5}$ (Ar ions).](image)

**4. Effect of charge limits on temporal behavior of particle charging**

The particle charging time is defined as the time it takes to reach a steady-state particle charge distribution, starting from all neutral particles. Charging time is inversely proportional to the particle diameter and to ion density. Fig. 3 shows results of Monte Carlo simulations for the temporal evolution of average particle charge for 5 nm diameter particles without charge limit, or with charge limits of either 5 or 10, all for the same plasma conditions. The more severe the charge limit, the more rapidly particles reach a stationary charge distribution. In the example shown, 5 nm particles reach a steady-state average charge after a few ms, while average charge is still notably fluctuating at 100 ms for the case without charge limits. This feature may be important for plasma processes with short time scales for changes in conditions, such as pulsed plasmas. In such cases, the assumption of a stationary particle charge distribution may not be valid, and a finite-rate charging model must be used.

![Fig. 3. Temporal evolution of average particle charge for 5 nm particles with various charge limits. Same plasma conditions as Fig. 2.](image)

**5. Conclusions**

We derived an analytical expression for the stationary particle charge distribution in a dusty plasma, accounting for the existence of particle charge limits. Since particle charging in dusty plasmas is the key phenomenon that affects the plasma-nanoparticle coupling, one can expect the effect of charge limits on charge distributions to have a number of repercussions.

For example, the EEDF in plasmas can be strongly affected by the presence of charged nanoparticles. Collection and momentum cross sections for collisions of electrons with nanoparticles depend on the particle potential, and therefore the existence of particle charge limits may strongly affect the EEDF. We expect particle charge limits to increase the high-energy tail of the EEDF,
as the electron energy needed to overcome the particle potential is reduced. The work presented here assumes that electron energy distribution functions (EEDFs) are Maxwellian. However, we find that particle charge distributions are independent of the form of the EEDF in cases where particle charge is sufficiently limited [6].

Additionally, we expect particle charge limits to affect fluctuations in nanoparticle temperature, which play an important role in nanoparticle crystallization, because electron-ion recombination on particle surfaces is a key driver for these fluctuations [7].

6. Acknowledgments

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7. References