Numerical enhancement of the microwave cavity method for plasma density measurement

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Abstract: Microwave resonator method is well established and sensitive technique of plasma density measurements, especially in low pressure plasmas. The traditional Slater solution of the electromagnetic field perturbation (suitable for small perturbations only) is replaced with more accurate, yet comprehensible numerical model, also addressing further phenomena such as the resonator eigenmodes and the stability of the dominant mode.

Keywords: numerical model, resonator, microwave, Q-factor, plasma density

1. Introduction

The microwave cavity method is based on the shift of eigenfrequency of a partially plasma-filled resonator, originally described by Slater [1]. Due to low collision frequency it is mostly used in low pressure plasma diagnostic. Slater approach uses perturbation method, which is simple and analytically solvable but it introduces certain inconsistencies and the plasma induced perturbation must be small. In many plasma studies, the high permittivity of the discharge tube (fused silica permittivity is almost 4) can easily violate this condition as the plasma is confined nearby, i.e. in high field region. This can lead to significant errors (easily over 50%). The advances in numerical modelling permit direct calculation of electromagnetic fields and resonant frequencies, giving more accurate results over the traditional method.

This contribution is focused on the diagnostics of low pressure glow discharge sustained in air in a fused silica tube, extending our previous research [2]. The discharge tube passes axially through the cylindrical resonator closely coupled with Gunn diode [3]. The plasma density affects the self-oscillating frequency of the device. Computationally, the effect of various discharge radial profiles is also studied.

Apart from measurement results, the actual numerical modelling approach for the resonator method is investigated, suggesting an effective practice for model development in most set-ups. The pivot point of this study is a reliable prediction of the dominant resonator mode, a result of a Gunn diode properties and Q-factor for different modes.

2. Experimental set-up

The cylindrical resonator cavity with inner dimensions of 59.9 mm (diameter) and 21.0 mm (height) as shown in Fig. 1 can oscillate at frequencies as low as 3.8 GHz, the experiment however employs the TE_{110} mode at approx. 9.3 GHz, which is easily excited. Furthermore, the electromagnetic (EM) field distribution is convenient as it decreases towards the axis. This configuration is sensitive enough to detect even the smallest perturbations (such as the difference of air and vacuum in the tube) while keeping the EM field rather undistorted. This is important to avoid a spontaneous switching between resonator modes. The resonator openings for the discharge tube (10 mm o.d., 8 mm i.d.) are placed axially at both its bases. The resonator is magnetically coupled with a Gunn diode by a miniature loop placed at the centre of side wall and parallel to its bases. The negative differential resistance of the Gunn diode effectively un-dampens the resonator which leads to self oscillations at highest Q eigenfrequency of the resonator. The self-oscillating frequency is then detected by the frequency counter and sent into PC via serial cable.

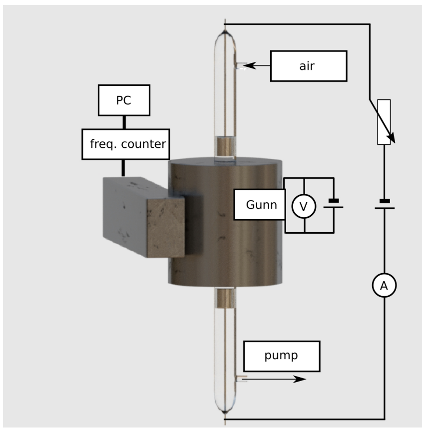

Fig. 1. The cylindrical microwave resonator set-up.

The discharge in air is realized in the discharge tube with 400-800 V of DC voltage applied to cylindrical electrodes. Rotary vane oil pump is used to maintain the system at 50 Pa while the air is admitted by needle valve.

3. Modelling

The modelling is done with the finite element method tool COMSOL Multiphysics and considers plasma as a dielectric medium using known relations for plasma permittivity [4]. In case of low pressure plasma, the collisions can usually be neglected and we get the real valued plasma permittivity (1).

$$\varepsilon_r = 1 - \frac{n_e e^2}{m_e \varepsilon_0 \omega^2} \tag{1}$$

where n_e denotes the plasma density, m_e mass of the electron, e elementary charge, ω the angular frequency of the probing electromagnetic wave and ε_0 the permittivity of vacuum.

With just a minor planning the modelling can be very straightforward. The desired model results are resonator eigenfrequencies along with the solved electromagnetic field and Q-factors computed for each mode. Initially let us discuss the differences of the analytical case (closed perfectly conductive cylindrical cavity with vacuum inside) vs. the real resonator deciding how much elaborate should the final model be.

The essential (and desired) difference from closed empty cavity is the discharge tube with the plasma inside. Associated with this is the presence of orifices leading into free space, which should be simulated by some non-reflective boundary condition (in our case by perfectly matched layers). On the other hand the Gunn diode coupling loop is small and placed in a region with weak EM field, and therefore it can be neglected. Finally, the conductivity of resonator walls is finite (and needs to be modelled as such). This concern may not seem necessary, since it is usually very high, but the finite value is needed to account for energy losses used in the Q-factor calculation.

In our opinion, it is a good practice to develop in iterations, beginning with a simple model, gradually incorporating more details. In Fig. 2 one simplified (starting) and one advanced model are shown.

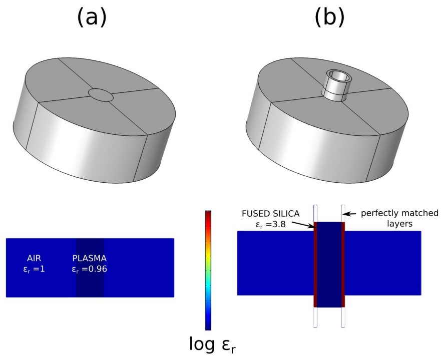

Fig. 2. The (a) simple and (b) advanced model geometry with colour coded permittivity shown in the rz-plane.

Following the standard COMSOL modelling routine (physics selection, set-up of the geometry, assignment of material properties to domains and boundaries and mesh creation) an appropriate numerical method is sought. For resonators with mostly closed geometry the *Eigenfrequency solver* with plasma permittivity parametric study worked the best. Input parameters are the centre frequency estimation and desired number of modes resolved. Raw results include eigenfrequency values and corresponding EM field distributions. From these the surface and volume integrals are computed, with the ratio being the Q-factor (Fig. 3).

This modelling approach can be carried out for other microwave resonators with not exceedingly open geometry (as the effectiveness of the eigenfrequency solver will drop). It should be noted, that some limitations may occur in different computational tools (such as inability to model finite conductivity boundaries, different types of solver) requiring further workarounds.

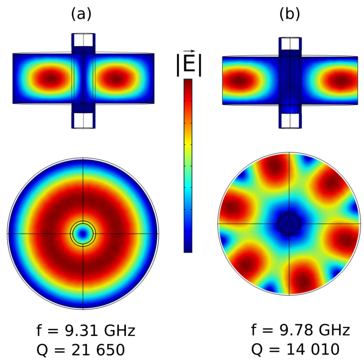

Fig. 3. The comparison of EM fields (rz-plane) and Q-factors for two computed modes around 9.3 GHz (the one on the left (a) being the used TE_{110} mode).

4. Results

The time (or frequency) measurements in physics are the most precise. The simple experiment described in this paper and operating near 10 GHz can easily resolve frequency shifts as low as 10 kHz. Furthermore, it turns out that the discharge tube presence is in fact advantageous and makes the phase shifts caused by plasma more pronounced by virtue of enhancing the EM field inside. Then, with the assistance of numerical results, the measured shift in eigenfrequency can be recalculated to absolute values of plasma density.

The simulated and the experimental results confirm the presence of a dominant and stable mode TE_{110}. The model also shows certain non-linearity in phase shift dependency on the plasma density, which is absent in the [1] and a scaling effect of different plasma density (radial) profiles.

5. Acknowledgements

This research was supported by the project LO1411 (NPU I) funded by the Ministry of Education Youth and Sports of the Czech Republic.

6. References

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