Thermodynamic Nonequilibrium Simulation of an Arc in Crossflow

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Abstract: Plasma applications such as wire-arc spraying and circuit breakers require understanding the interaction of an electric arc with a stream of cold gas flow perpendicular to the arc. A canonical arc in crossflow is simulated using a three-dimensional time-dependent thermodynamic nonequilibrium plasma flow model. The obtained shape of the arc, electron and heavy-species temperature distributions, and degree of thermodynamic nonequilibrium with varying total current are discussed.

Keywords: plasma-flow interaction, thermal plasma, thermodynamic nonequilibrium.

1. Introduction

Industrial plasma applications such as wire arc spraying, low voltage circuit breakers, as well as plasma arc tunnels for re-entry vehicle testing, involve the interaction of an electric arc with a stream of cold gas flow perpendicular to the arc. The so-called ‘arc in crossflow’ is a canonical plasma configuration whose study can provide important physical understanding directly relevant to those applications as well as elucidate fundamental plasma phenomena [1].

Figure 1 shows schematically the arc in crossflow. The system consists of direct-current (dc) cathode and anode electrodes enclosed within confining parallel walls separated by a distance $H$. Arc plasma is formed when an imposed electric field is established between the electrodes, permitting the transfer of a total amount of electric current $I_{\text{arc}}$. The plasma is subjected to a stream of gas parallel to the walls with mean axial velocity $U_i$ at the inlet. The gas produces convective cooling and drag on the plasma, forcing the plasma column to bend and causing an afterglow of ionized gas downstream from the electrodes. The system mainly depends on the type of gas, the inter-electrode spacing $H$, total current $I_{\text{arc}}$, and inflow velocity $U_i$.

![Fig. 1. Schematic of an arc in crossflow system with an inter-electrode distance $H$, showing the plasma attachment to the electrodes, the current path, and the nonequilibrium afterglow plasma.](image)

Due to their marked relevance to thermal plasma applications, the arc in crossflow has been extensively studied by both, experimental and computational means. Benenson et al [2] used the integrated line emission coefficient technique to determine the arc in cross flow stability and obtained the radial temperature distribution for different current inputs. Maeckar et al [3] developed an analytical model to explain the arc formation and bending. Kelkar et al [1] developed a three-dimensional (3D) arc in air cross flow model used to investigate the effect of variations in current and electrode spacing on the temperature, current density, and axial velocity fields. Lincun et al [4] developed a 3D arc in crossflow model for argon, which was used for a parametric study based on axial velocity, current electrode spacing.

All previous computational investigations of the arc in crossflow have relied on models based on the Local Thermodynamic Equilibrium (LTE) assumption. The LTE assumption implies that the heavy species (molecules, atoms, ions) are in kinetic equilibrium with electrons, and hence both can be characterized by a single equilibrium temperature $T$. The LTE assumption, often used to describe thermal plasmas such as atmospheric-pressure arcs, is valid within the core of the plasma, but is often invalid in the plasma periphery, especially when the plasma interacts strongly with its environment. The latter is particularly expected for an arc in crossflow, especially for high values of $U_i$ and/or low vales of $I_{\text{arc}}$. In spite of the relevance of thermodynamic nonequilibrium, there have been no reports on the analysis of the arc in crossflow using non-LTE (NLTE) plasma models. NLTE models can describe the interdependence between heavy-species and electron energy, for example, given by the evolution of the distributions of heavy-species temperature $T_h$ and electron temperature $T_e$. An NLTE model can provide novel fundamental insight of the arc in crossflow, particularly of the dynamics occurring in regions near the electrodes and in the region surrounding the arc, where nonequilibrium is expected to be predominant.

This paper presents simulation results of an argon arc in crossflow using for the first time a time-dependent, 3D, chemical equilibrium and thermodynamic
nonequilibrium (NLTE) model. The numerical formulation of the model is based on a monolithic approach that treats the fluid and electromagnetic equations in a fully-coupled manner using a Variational Multiscale Finite Element Method (VMS-FEM).

2. Modeling of arc in cross flow

2.1. Assumptions

- The plasma is treated as a compressible, reactive electromagnetic fluid in chemical equilibrium and thermodynamic nonequilibrium (NLTE).
- The plasma is considered to be quasi-neutral, non-magnetized, and non-relativistic.
- Charge transport is dominated by the electric field distribution and electron diffusion; ion diffusion and Hall currents are assumed negligible.
- The plasma is considered to be optically thin.

2.2. Model equations

Consisting with the prior assumptions, the set of equations describing the plasma flow are constituted by: mass conservation, mass-average momentum conservation, conservation of thermal energy of heavy-species and of electrons, electric charge and magnetic induction conservation [5]. These equations form a single set of transient-advective-diffusive-reactive (TADR) transport equations listed in Table 1.

Table 1: Set of fluid-electromagnetic evolution equations for the NLTE plasma flow model. For each equation: Transient + Advective – Diffusive – Reactive = 0.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Transient</th>
<th>Adve cessive</th>
<th>Diffusive</th>
<th>Reactive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conservation of total mass</td>
<td>$\frac{\partial}{\partial t} \rho$</td>
<td>$\rho u \cdot \nabla \rho$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Conservation of linear momentum</td>
<td>$\rho \frac{\partial}{\partial t} u$</td>
<td>$\rho u \cdot \nabla u$</td>
<td>$\nabla \cdot (\kappa_{tr} \nabla T_{h})$</td>
<td>$\frac{\partial}{\partial t} p_{h} + u \cdot \nabla p_{h} + K_{ch}(T_{e} - T_{h}) - \tau : \nabla u$</td>
</tr>
<tr>
<td>Thermal energy of heavy species</td>
<td>$\rho \frac{\partial}{\partial t} h_{h}$</td>
<td>$\rho u \cdot \nabla h_{h}$</td>
<td>$\nabla \cdot (\kappa_{oe} \nabla T_{e})$</td>
<td>$\frac{\partial}{\partial t} p_{e} + u \cdot \nabla p_{e} - K_{ch}(T_{e} - T_{h}) - 4\pi e_{e}^{2} + J_{q} \cdot (E + u \times B)$</td>
</tr>
<tr>
<td>Thermal energy of electrons</td>
<td>$\rho \frac{\partial}{\partial t} e_{e}$</td>
<td>$\rho u \cdot \nabla e_{e}$</td>
<td>$\nabla \cdot (\alpha\nabla \phi_{e}) - \nabla \cdot (\alpha u \times (\nabla \times A))$</td>
<td>0</td>
</tr>
<tr>
<td>Charge conservation</td>
<td>0</td>
<td>0</td>
<td>$\nabla \cdot (\alpha\nabla \phi_{e}) - \nabla \cdot (\alpha u \times (\nabla \times A))$</td>
<td>0</td>
</tr>
<tr>
<td>Magnetic induction</td>
<td>$\mu_{0} \alpha \frac{\partial}{\partial t} A$</td>
<td>$\mu_{0} \alpha \nabla \phi_{e}$</td>
<td>$\nabla \times A$</td>
<td>0</td>
</tr>
</tbody>
</table>

In Table 1, $\frac{\partial}{\partial t}$ represents the partial derivative with respect to time, $\nabla$ and $\nabla \cdot$ represents the gradient and divergence operators, respectively; $\rho$ is mass density, $p$ pressure, $h_{h}$ and $h_{e}$ are the enthalpies of electrons and heavy-species, respectively; $u$ represents mass-averaged velocity, $\mu$ is the dynamic viscosity, $T$ is the transpose operator, $\delta$ is the Kronecker delta tensor. The momentum and thermal energy conservation equations implicitly represent mass conservation, mathematically $\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho u) = 0$. The diffusive term in the electron thermal energy conservation gives the transport of electrons due to the characteristic time for inter-particle collision [5].

The set of equations in Table 1 can be expressed in residual form for the vector of unknowns $Y$ as:

$$R(Y) = A_{b} \frac{\partial}{\partial t} Y + (A_{d} \frac{\partial}{\partial t}) Y - \frac{\partial}{\partial t} (K_{d} \frac{\partial}{\partial t} Y) - (S_{d} Y - S_{b}) = 0,$$

(1)

Where $A_{b}$, $A_{d}$, $K_{d}$, $S_{d}$, $S_{b}$ are coefficient matrices that are used to characterize the different transport processes, $i$ and $j$ are spatial indices, and Einstein’s
convention of repeated indexes has been used. The system in Eq. (1) is solved for the set of prime variables given by:

\[ \mathbf{Y} = [p \ u \ T_h \ T_e \ \phi_p \ \mathbf{A}] \tag{2} \]

2.3. Numerical method

The inherent advantages of Finite Element Methods (FEM) have led to their extensive use in diverse fields, including plasma flow modeling. Among FEMs, the Variational Multiscale method (VMS) has demonstrated to provide a general and robust formulation for diverse types of problems, such as scalar transport, incompressible, compressible, reactive and turbulent flows, radiation transport, and magnetohydrodynamic flows. The VMS method has also been proven to be effective for the modeling plasma flows, which are highly nonlinear [6]. The VMS method starts with a Galerkin FEM formulation of the problem in Eq. (1), and then on decomposing it into two sub-problems: one for the large scales, which can be captured by the discretization, and another one for the small scales, which is parametrically solved in terms of the large scales. A detailed description of the VMS formulation used is found in [6].

3. Simulation set-up

The geometrical set-up for the arc in cross flow is shown in Fig. 2, which depicts the physical domain, boundary sides, and plasma column. The set of boundary conditions used is listed in Table 2. The no-slip condition is applied at electrodes and the wall surrounding them (Anode, Cathode, Wall). Gas flow at the inflow is specified by a parabolic velocity profile \( u_0(x) \). At the Outflow_y, the atmospheric pressure condition is applied, while zero normal gradient is used at the Outflow_x boundary.

![Fig. 2. Physical domain and boundary conditions.](image)

The current density \( J_{\text{cath}} \) applied at the Cathode boundary follows a Gaussian distribution, which provides a total value of current \( I_{\text{tot}} \) to the system. Four set of imposed currents 17 [A], 25 [A], 30 [A] and 34 [A] are considered in the simulations. The cathode temperature \( T_c \) is taken to be closer to the melting point of Tungsten, similarly as used in [2].

In Table 2, \( p_0 = 1.01325 \cdot 10^5 \) [Pa] (atmospheric pressure), \( u_0 = [0 \ 0 \ U_x] = 0.5 \) [ms\(^{-1}\)], \( T_0 \) and \( T_w = 500 \) [K], \( T_c \) - Gaussian distribution from 500 to 3000 [K]. The geometrical dimensions \( L_x \times L_y \times L_z \) are 6.64 [mm] x 14 [mm] x 25.3 [mm].

<table>
<thead>
<tr>
<th>Boundary</th>
<th>( p )</th>
<th>( u )</th>
<th>( T_h )</th>
<th>( T_e )</th>
<th>( \phi_p )</th>
<th>( \mathbf{A} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflow</td>
<td>( \partial_s p = 0 )</td>
<td>( u = u_0 )</td>
<td>( T_h = T_0 )</td>
<td>( T_e = T_0 )</td>
<td>( \partial_s \phi_p = 0 )</td>
<td>( \mathbf{A} = 0 )</td>
</tr>
<tr>
<td>Anode</td>
<td>( \partial_s p = 0 )</td>
<td>( u = 0 )</td>
<td>( -k_0 \frac{\partial T_h}{\partial n} = h_u(T_h - T_w) )</td>
<td>( \partial_s T_e = 0 )</td>
<td>( \partial_s \phi_p = 0 )</td>
<td>( \partial_s \mathbf{A} = 0 )</td>
</tr>
<tr>
<td>Cathode</td>
<td>( \partial_s p = 0 )</td>
<td>( u = 0 )</td>
<td>( T_h = T_c )</td>
<td>( \partial_s T_e = 0 )</td>
<td>( -\sigma \frac{\partial \phi_p}{\partial n} = J_{\text{cath}} )</td>
<td>( \partial_s \mathbf{A} = 0 )</td>
</tr>
<tr>
<td>Wall</td>
<td>( \partial_s p = 0 )</td>
<td>( u = 0 )</td>
<td>( T_h = T_w )</td>
<td>( \partial_s T_e = 0 )</td>
<td>( \partial_s \phi_p = 0 )</td>
<td>( \partial_s \mathbf{A} = 0 )</td>
</tr>
<tr>
<td>Outflow_x</td>
<td>( p = p_w )</td>
<td>( \partial_n u = 0 )</td>
<td>( \partial_n T_h = 0 )</td>
<td>( \partial_n T_e = 0 )</td>
<td>( \partial_n \phi_p = 0 )</td>
<td>( \partial_n \mathbf{A} = 0 )</td>
</tr>
<tr>
<td>Outflow_y</td>
<td>( \partial_s p = 0 )</td>
<td>( \partial_n u = 0 )</td>
<td>( \partial_n T_h = 0 )</td>
<td>( \partial_n T_e = 0 )</td>
<td>( \partial_n \phi_p = 0 )</td>
<td>( \partial_n \mathbf{A} = 0 )</td>
</tr>
</tbody>
</table>

4. Results

Figure 3 summarizes the effect of imposed current on the distributions of heavy species temperature \( T_h \), the nonequilibrium parameter \( \theta \) (\( \theta = T_e / T_h \)), and the axial velocity \( u_z \). The maximum heavy species temperature varies from 14 [KK] for 17 [A] of imposed current up to 25 [KK] for 34 [A] of imposed current. Similarly, the maximum axial velocity varies from 90 [ms\(^{-1}\)] to 450 [ms\(^{-1}\)]. The imposed current increases the overall system temperature due to Joule heating. Due to the constricted cathode arc attachment and diffuse anode attachment, the temperature at the cathode vicinity is relatively higher. This in turn results in an asymmetrical velocity distribution, manifested by the formation of a cathode jet. The nonequilibrium parameter \( \theta \) shows minimal variation throughout most of the plasma column, except upstream. This indicates that under the studied conditions, the plasma is mostly in a LTE state. However, the variation from LTE state...
to NLTE is very abrupt near the walls and electrodes. Also, as the current is increased, the arc shape changes from bow-shaped to cusp-shaped. This was also experimentally observed by Benenson et al. [2].

A dimensionless analysis helps to understand the effect of varying the model parameters. Specifically, for the arc in cross flow, the main parameters (electrode gap, inflow velocity, and imposed current) can be combined to form the dimensionless number $\Pi_h$, often referred as the ‘enthalpy number’, and defined in Eq. 3 [7]. In Eq. 3, $r$ denotes a reference value of the given property evaluated at the reference temperature stated in [7] for the arc in cross flow system.

$$\Pi_h = \frac{\sigma_h \rho U H^3}{I_{tot}}.$$  \hspace{1cm} (3)

The parameter $\Pi_h$ represents the relative strength of the imposed flow over the arc. Therefore, low values of $\Pi_h$ indicate that the arc is relatively unaffected by the imposed flow. In the studied simulations, $\Pi_h$ varies from 0.3 for 17 [A] to 0.075 for 34 [A], which may explain the observed minimal deviation from LTE.

Fig. 3. Isosurfaces of heavy species temperature $T_s$ (left), nonequilibrium parameter $\theta = T_r/T_h$ (center), and axial velocity $u_z$ (right) for imposed currents (a) 17 [A], (b) 25 [A], (c) 30 [A], and (d) 34 [A] depicting the variation in arc shape from bow- to cusp-shape, the predominance of nonequilibrium upstream of the arc, and the cathode jet.

5. Conclusion

The arc in crossflow is a canonical arc discharge relevant to diverse applications, such wire arc spraying and circuit breakers. The heavy-species temperature, degree of thermodynamic nonequilibrium, and velocity distribution for an argon arc in cross flow are investigated using a three-dimensional thermodynamic nonequilibrium (NLTE) model. The model is numerically solved using a VMS-FEM method. The results show the transition from bow-shaped to cusp-shaped arc for increasing value of current. The nonequilibrium effects show minimal variation except for an abrupt change at the electrode vicinity. The obtained behaviour can be partially explained by the relatively low values of the dimensionless enthalpy number, which characterizes the relative strength of the interaction between the imposed flow over the arc.

Acknowledgements

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References